

31.10 1)

Περιορισμοί : i) $x > 0$

$$\Rightarrow \log(4x)^{\log 2 + \log \sqrt{x}} = \log 1 \Rightarrow (\log 2 + \log \sqrt{x}) \cdot \log 4x = 0 \Rightarrow$$

$$\Rightarrow \log \sqrt{x} + \log 2 = 0 \quad \text{ή} \quad \log 4x = 0$$

$$\Rightarrow \log(2\sqrt{x}) = \log 1 \quad \text{ή} \quad \log 4x = \log 1$$

$$\Rightarrow 2\sqrt{x} = 1 \quad \text{ή} \quad 4x = 1$$

$$\Rightarrow x = \frac{1}{4}$$

31.10 2)

$$2^{\log x} + 3 \cdot 4^{\log x} = 52$$

Περιορισμοί : i) $x > 0$

$$\Rightarrow 2 \log^x + 3 \cdot 2^{2 \log x} - 52 = 0$$

θέτω $2^{\log x} = \omega > 0$ και η εξίσωση γράφεται :

$$\omega + 3\omega^2 - 52 = 0 \Rightarrow 3\omega^2 + \omega - 52 = 0$$

$$\Delta = 625 \quad \omega_{1,2} = \frac{-1 \pm 25}{6} \begin{cases} \nearrow x_1 = 4 \\ \searrow x_2 = -\frac{13}{3} < 0 \text{ απορρίπτεται} \end{cases}$$

$$\text{Άρα : } \omega = 4 \Rightarrow 2^{\log x} = 2^2 \Rightarrow \log x = 2 \Rightarrow x = 100$$

31.10 3)

περιορισμοί : $x > 0$

$$\ln\left(\frac{x}{2}\right) = \ln \sqrt{x} \Rightarrow \frac{x}{2} = \sqrt{x} \xrightarrow{x>0} \frac{x^2}{4} = x \Rightarrow$$

$$\Rightarrow x\left(\frac{x}{4} - 1\right) = 0 \Rightarrow$$

$$\Rightarrow x = 0 \text{ απορρίπτεται} \quad \text{ή} \quad x = 4 \quad \text{δεκτή}$$

31.10 4)

Περιορισμοί : i) $x > 0$

$$\text{ii) } \log \sqrt{x} \neq 0 \Rightarrow x \neq 1$$

$$2 \log x + \frac{2}{\frac{1}{2} \log x} = 6 \Rightarrow$$

$$\Rightarrow 2 \log x + \frac{4}{\log x} = 6 \Rightarrow \log x + \frac{2}{\log x} = 3$$

θέτω $\log x = \omega \neq 0$ και η εξίσωση γράφεται :

$$\omega + \frac{2}{\omega} = 3 \xrightarrow{\omega \neq 0} \omega^2 + 2 - 3\omega = 0 \Rightarrow \omega^2 - 3\omega + 2 = 0$$

$$\Delta = 1 \quad \omega_{1,2} = \frac{3 \pm 1}{2} \begin{matrix} \nearrow \omega_1 = 2 \\ \searrow \omega_2 = 1 \end{matrix}$$

Άρα

$$\omega = 2 \Rightarrow \log x = 2 \Rightarrow x = 10^2 \quad \text{δεκτή} \quad \text{ή} \quad \omega = 1 \Rightarrow \log x = 1 \Rightarrow x = 10 \quad \text{δεκτή}$$