

ΘΕΜΑ 1

α) Λ β) Σ γ) Σ δ) Σ ε) Σ

ΘΕΜΑ 2

$$\left(\frac{1}{2}\right)^{x-6} \geq 2^{3x+2} \Rightarrow (2^{-1})^{x-6} \geq 2^{3x+2} \Rightarrow$$

$$\Rightarrow 2^{6-x} \geq 2^{3x+2} \Rightarrow 6-x \geq 3x+2 \Rightarrow 4x \leq 4 \Rightarrow x \leq 1$$

ΘΕΜΑ 3

$$x + \log(2^x + 1) = x \log 5 + \log 6 \Rightarrow$$

$$\Rightarrow x(1 - \log 5) = \log 6 - \log(2^x + 1) \Rightarrow$$

$$\Rightarrow x(\log 10 - \log 5) = \log\left(\frac{6}{2^x + 1}\right) \Rightarrow$$

$$\Rightarrow x \log\left(\frac{10}{5}\right) = \log\left(\frac{6}{2^x + 1}\right) \Rightarrow$$

$$\Rightarrow x \log 2 = \log\left(\frac{6}{2^x + 1}\right) \Rightarrow$$

$$\Rightarrow \log 2^x = \log\left(\frac{6}{2^x + 1}\right) \Rightarrow$$

$$\Rightarrow 2^x = \frac{6}{2^x + 1} \Rightarrow$$

$$\Rightarrow 2^{2x} + 2^x = 6 \Rightarrow$$

$$\Rightarrow 2^{2x} + 2^x - 6 = 0$$

Θέτουμε $y = 2^x$ και έχουμε : $y^2 + y - 6 = 0$

$$\Delta = 25 \quad y_{1,2} = \frac{-1 \pm 5}{2} \begin{matrix} \nearrow 2 \\ \searrow -3 \end{matrix}$$

Για $y = 2$, $2^x = 2 \Rightarrow x = 1$

Για $y = -3$, $2^x = -3$ αδύνατη

Άρα η λύση της εξίσωσης είναι η $x = 1$

ΘΕΜΑ 4

Χρησιμοποιούμε τον τύπο αλλαγής βάσης :

α' μέλος =

$$= \frac{1}{\log_1 1 + \log_2 2 + \dots + \log_2 \kappa} + \frac{1}{\log_3 1 + \log_3 2 + \dots + \log_3 \kappa} + \frac{1}{\log_\kappa 1 + \log_\kappa 2 + \dots + \log_\kappa \kappa} =$$

$$\begin{aligned}
&= \frac{1}{\frac{\log 1}{\log 2} + \frac{\log 2}{\log 2} + \dots + \frac{\log \kappa}{\log 2}} + \frac{1}{\frac{\log 1}{\log 3} + \frac{\log 2}{\log 3} + \dots + \frac{\log \kappa}{\log 3}} + \dots + \frac{1}{\frac{\log 1}{\log \kappa} + \frac{\log 2}{\log \kappa} + \dots + \frac{\log \kappa}{\log \kappa}} = \\
&= \frac{1}{\log(1 \cdot 2 \cdots \kappa)} + \frac{1}{\log(1 \cdot 2 \cdots \kappa)} + \dots + \frac{1}{\log(1 \cdot 2 \cdots \kappa)} = \\
&= \frac{\log 2 + \log 3 + \dots + \log \kappa}{\log(1 \cdot 2 \cdots \kappa)} = \frac{\log(2 \cdots \kappa)}{\log(1 \cdot 2 \cdots \kappa)} = 1 = \beta' \text{ μέλος}
\end{aligned}$$