

31.9

α) Πρέπει $\alpha + 2 \neq 0$ και $\frac{3\alpha}{\alpha+2} > 0 \Rightarrow 3\alpha(\alpha+2) > 0$ $\frac{-2}{+} \mid \frac{0}{-} \mid + \Rightarrow \alpha < -2$ ή $\alpha > 0$

Άρα $\alpha \in (-\infty, -2) \cup (0, +\infty)$

β) Πρέπει $\alpha + 2 \neq 0$ και

$$\frac{3\alpha}{\alpha+2} > 1 \Rightarrow \frac{3\alpha}{\alpha+2} - 1 > 0 \Rightarrow \frac{3\alpha - \alpha - 2}{\alpha+2} > 0 \Rightarrow \frac{2(\alpha-1)}{\alpha+2} > 0 \Rightarrow$$

$$\Rightarrow (\alpha-1)(\alpha+2) > 0 \quad \frac{-2}{+} \mid \frac{1}{-} \mid + \Rightarrow \alpha < -2 \text{ ή } \alpha > 1$$

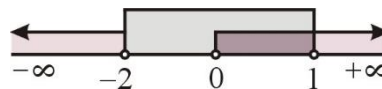
Άρα $\alpha \in (-\infty, -2) \cup (1, +\infty)$

γ) Πρέπει $\alpha + 2 \neq 0$ και $0 < \frac{3\alpha}{\alpha+2} < 1 \Rightarrow$

$$\Rightarrow \frac{3\alpha}{\alpha+2} > 0 \quad \text{και} \quad \frac{3\alpha}{\alpha+2} < 1 \Rightarrow$$

$$\Rightarrow \alpha \in (-\infty, -2) \cup (0, +\infty) \text{ και } \alpha \in (-2, 1)$$

$$\Rightarrow \alpha \in (0, 1)$$



31.10 1)

Περιορισμοί: i) $x > 0$

$$\Rightarrow \log(4x)^{\log 2 + \log \sqrt{x}} = \log 1 \Rightarrow (\log 2 + \log \sqrt{x}) \cdot \log 4x = 0 \Rightarrow$$

$$\Rightarrow \log \sqrt{x} + \log 2 = 0 \quad \text{ή} \quad \log 4x = 0$$

$$\Rightarrow \log(2\sqrt{x}) = \log 1 \quad \text{ή} \quad \log 4x = \log 1$$

$$\Rightarrow 2\sqrt{x} = 1 \quad \text{ή} \quad 4x = 1$$

$$\Rightarrow x = \frac{1}{4}$$

31.10 2)

$$2^{\log x} + 3 \cdot 4^{\log x} = 52$$

Περιορισμοί: i) $x > 0$

$$\Rightarrow 2\log^x + 3 \cdot 2^{2\log x} - 52 = 0$$

θέτω $2^{\log x} = \omega > 0$ και η εξίσωση γράφεται :

$$\omega + 3\omega^2 - 52 = 0 \Rightarrow 3\omega^2 + \omega - 52 = 0$$

$$\Delta = 625 \quad \omega_{1,2} = \frac{-1 \pm 25}{6} \begin{matrix} \nearrow x_1 = 4 \\ \searrow x_2 = -\frac{13}{3} < 0 \end{matrix} \quad \text{απορρίπτεται}$$

$$\text{Άρα : } \omega = 4 \Rightarrow 2^{\log x} = 2^2 \Rightarrow \log x = 2 \Rightarrow x = 100$$

31.10 3)

περιορισμοί : $x > 0$

$$\ln\left(\frac{x}{2}\right) = \ln \sqrt{x} \Rightarrow \frac{x}{2} = \sqrt{x} \xrightarrow{x>0} \frac{x^2}{4} = x \Rightarrow$$

$$\Rightarrow x\left(\frac{x}{4} - 1\right) = 0 \Rightarrow$$

$$\Rightarrow x = 0 \quad \text{απορρίπτεται} \quad \text{ή} \quad x = 4 \quad \text{δεκτή}$$

31.10 4)

Περιορισμοί : i) $x > 0$

$$\text{ii) } \log \sqrt{x} \neq 0 \Rightarrow x \neq 1$$

$$2\log x + \frac{2}{\frac{1}{2}\log x} = 6 \Rightarrow$$

$$\Rightarrow 2\log x + \frac{4}{\log x} = 6 \Rightarrow \log x + \frac{2}{\log x} = 3$$

θέτω $\log x = \omega \neq 0$ και η εξίσωση γράφεται :

$$\omega + \frac{2}{\omega} = 3 \xrightarrow{\omega \neq 0} \omega^2 + 2 - 3\omega = 0 \Rightarrow \omega^2 - 3\omega + 2 = 0$$

$$\Delta = 1 \quad \omega_{1,2} = \frac{3 \pm 1}{2} \begin{matrix} \nearrow \omega_1 = 2 \\ \searrow \omega_2 = 1 \end{matrix}$$

Άρα

$$\omega = 2 \Rightarrow \log x = 2 \Rightarrow x = 10^2 \quad \text{δεκτή} \quad \text{ή} \quad \omega = 1 \Rightarrow \log x = 1 \Rightarrow x = 10 \quad \text{δεκτή}$$