31.8

$$f(x)=\frac{1}{2}\left(\alpha^{x}+\alpha^{-x}\right) \quad , \quad g(x)=\frac{1}{2}\left(\alpha^{x}-\alpha^{-x}\right)$$

Είναι : $f(x+y)=\frac{1}{2}\left(\alpha^{x+y}+\alpha^{-x-y}\right)=\frac{1}{2}\left(\alpha^{x}\cdot\alpha^{y}+\alpha^{-x}\cdot\alpha^{-y}\right)$

και : $f(x)\cdot f(y)+g(x)\cdot g(y)=$

$$=\frac{1}{2}\left(\alpha^{x}+\alpha^{-x}\right)\cdot\frac{1}{2}\left(\alpha^{y}+\alpha^{-y}\right)+\frac{1}{2}\left(\alpha^{x}-\alpha^{-x}\right)\cdot\frac{1}{2}\left(\alpha^{y}-\alpha^{-y}\right)=$$

$$=\frac{1}{4}\left(\alpha^{x+y}+\alpha^{x-y}+\alpha^{-x+y}+\alpha^{-x-y}\right)+\frac{1}{4}\left(\alpha^{x+y}-\alpha^{x-y}-\alpha^{-x+y}+\alpha^{-x-y}\right)=$$

$$=\frac{1}{4}\alpha^{x+y}+\cancel{\frac{1}{4}\alpha^{x-y}}+\cancel{\frac{1}{4}\alpha^{-x+y}}+\frac{1}{4}\alpha^{-x-y}+\frac{1}{4}\alpha^{x+y}-\cancel{\frac{1}{4}\alpha^{x-y}}-\cancel{\frac{1}{4}\alpha^{-x+y}}+\frac{1}{4}\alpha^{-x-y}=$$

$$=\frac{1}{2}\alpha^{x+y}+\frac{1}{2}\alpha^{-x-y}=\frac{1}{2}\left(\alpha^{x}\cdot\alpha^{y}+\alpha^{-x}\cdot\alpha^{-y}\right)=f(x+y)$$