

31.5

Έχουμε ισοδύναμα

$$\frac{\log \alpha + \log \beta}{2} \leq \log \frac{\alpha + \beta}{2} \Leftrightarrow \log \alpha + \log \beta \leq 2 \log \frac{\alpha + \beta}{2} \Leftrightarrow \log \alpha \beta \leq \log \left(\frac{\alpha + \beta}{2} \right)^2 \Leftrightarrow$$

$$\Leftrightarrow \alpha \beta \leq \left(\frac{\alpha + \beta}{2} \right)^2 \Leftrightarrow \alpha \beta \leq \frac{(\alpha + \beta)^2}{4} \Leftrightarrow 4\alpha\beta \leq (\alpha + \beta)^2 \Leftrightarrow 4\alpha\beta \leq \alpha^2 + 2\alpha\beta + \beta^2 \Leftrightarrow$$

$$\Leftrightarrow 0 \leq \alpha^2 - 2\alpha\beta + \beta^2 \Leftrightarrow 0 \leq (\alpha - \beta)^2 \text{ που ισχύει}$$