

31.17 1)

$$8^x + 18^x = 2 \cdot 27^x \Rightarrow 2^{3x} + 2^x \cdot 9^x = 2 \cdot 3^{3x} \Rightarrow$$

$$\Rightarrow \frac{2^{3x}}{2^x \cdot 9^x} + 1 = \frac{2 \cdot 3^{3x}}{2^x \cdot 9^x} \Rightarrow \frac{2^{2x}}{3^{2x}} + 1 = 2 \cdot \frac{3^x}{2^x}$$

Θέτω $\left(\frac{2}{3}\right)^x = \omega$ και η εξίσωση γράφεται :

$$\omega^2 + 1 = \frac{2}{\omega} \Rightarrow \omega^3 + \omega = 2 \Rightarrow \omega^3 + \omega - 2 = 0$$

Η $\omega = 1$ είναι ρίζα

Σχήμα Horner :

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

Άρα $\omega^3 + \omega - 2 = 0 \Rightarrow (\omega - 1)(\omega^2 + \omega + 2) = 0 \Rightarrow \omega = 1$ ή $\omega^2 + \omega + 2 = 0 \Rightarrow \Delta = -7 < 0$ αδύνατη

Άρα $\omega = 1 \Rightarrow \left(\frac{2}{3}\right)^x = 1 \Rightarrow \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^0 \Rightarrow x = 0$

31.17 2)

$$2^{x^2+x-6} - 2^{x^2+x-9} = 56 \Rightarrow 2^{x^2+x-9} \cdot 2^3 - 2^{x^2+x-9} = 56$$

Θέτω $2^{x^2+x-9} = \omega$ και η εξίσωση γράφεται :

$$8\omega - \omega = 56 \Rightarrow 7\omega = 56 \Rightarrow \omega = 8$$

Άρα

$$2^{x^2+x-9} = 8 \Rightarrow 2^{x^2+x-9} = 2^3 \Rightarrow x^2 + x - 9 = 3 \Rightarrow x^2 + x - 12 = 0 \Rightarrow \Delta = 49, x_{1,2} = \frac{-1 \pm 7}{2} \begin{matrix} \nearrow x_1 = 3 \\ \searrow x_2 = -4 \end{matrix} \Rightarrow x = -4 \text{ ή } x = 3$$

31.17 3)

Περιορισμός : $x > 0$

Με αυτό τον περιορισμό έχουμε

$$\left(\frac{\log x}{2}\right)^{\log^2 x + \log x - 2} = \log \sqrt{x} \Rightarrow \left(\frac{\log x}{2}\right)^{\log^2 x + 2\log x - 2} = \frac{\log x}{2} \Rightarrow \left(\frac{\log x}{2}\right)^{\log^2 x + 2\log x - 3} = 1 \Rightarrow$$

$$\Rightarrow \log \left[\frac{\log x}{2} \right]^{\log^2 x + 2\log x - 3} = \log 1 \Rightarrow (\log^2 x + 2\log x - 3) \log \left(\frac{\log x}{2} \right) = 0 \Rightarrow$$

$$\Rightarrow \log^2 x + 2\log x - 3 = 0$$

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$$\log \left(\frac{\log x}{2} \right) = 0 \Rightarrow$$

$$\begin{array}{ll} \omega = -3 & \text{ή} \quad \omega = 1 \\ \Rightarrow \log x = -3 & \Rightarrow \log x = 1 \\ \Rightarrow x = 1000 \text{ δεκτή} & \Rightarrow x = 10 \text{ δεκτή} \end{array}$$

31.17 4)

$$\left. \begin{array}{l} 10^{x-1} + 2 > 0 \quad \text{ισχύει} \\ \log(1 + \log(10^{x-1} + 2)) > 0 \quad \text{ισχύει} \\ x > 0 \end{array} \right\} \Rightarrow x > 0$$

Με αυτόν τον περιορισμό έχουμε

$$\log[\log(10^{x-1} + 2) + 1] = \log 2 + \log x \Rightarrow$$

$$\Rightarrow \log(\log(10^{x-1} + 2) + 1) = \log 2x \Rightarrow \log(10^{x-1} + 2) + 1 = 2x \Rightarrow$$

$$\Rightarrow \log(10^{x-1} + 2) = 2x - 1 \Rightarrow 10^{x-1} + 2 = 10^{2x-1} \Rightarrow$$

$$\Rightarrow \frac{10^x}{10} + 2 = \frac{10^{2x}}{10} \Rightarrow 10^{2x} - 10^x - 20 = 0$$

$$\text{Έστω } \omega = 10^x > 0 \Rightarrow \omega^2 - \omega - 20 = 0$$

$$\Delta = 81 \quad \omega_{1,2} = \frac{1 \pm 9}{2} \begin{array}{l} \nearrow \omega_1 = 5 \\ \searrow \omega_2 = -4 < 0 \end{array} \quad \text{απορρίπτεται}$$

$$\text{Άρα } \omega = 5 \Rightarrow 10^x = 5 \Rightarrow x = \log 5 \text{ δεκτή}$$