31.12 1)

$$(\Sigma):\begin{cases}\log x-\log y=\log 3\\ \log(x-y)=1\end{cases}$$

Περιορισμοί : $\begin{cases}x>0\\ y>0\\ x-y>0\end{cases}$

$$(\Sigma):\Rightarrow\begin{cases}\log\left(\frac{x}{y}\right)=\log 3\\ \log(x-y)=\log 10\end{cases}\Rightarrow\begin{cases}\frac{x}{y}=3\\ x-y=10\end{cases}\Rightarrow\begin{cases}x=3y\\ 3y-y=10\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}x=3y\\ 2y=10\end{cases}\Rightarrow\begin{cases}x=15\\ y=5\end{cases}\Rightarrow(x\ ,\ y)=(15\ ,\ 5)$$ δεκτή

31.12 2)

$$(\Sigma):\begin{cases}x^y=y^x\\ 3^x=9^y\end{cases}\overset{x,y>0}{\Rightarrow}\begin{cases}\log x^y=\log y^x\\ \log 3^x=\log 9^y\end{cases}\Rightarrow\begin{cases}y\cdot\log x=x\cdot\log y\\ x\cdot\log 3=y\cdot 2\log 3\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}y\log 2y=2y\log y\\ x=2y\end{cases}\overset{y\neq 0}{\Rightarrow}\begin{cases}\log 2y=\log y^2\\ x=25\end{cases}\Rightarrow\begin{cases}y^2=2y\\ x=2y\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}y(y-2)=0\\ x=2y\end{cases}\overset{y\neq 0}{\Rightarrow}\begin{cases}y=2\\ x=4\end{cases}\Rightarrow(x\ ,\ y)=(4\ ,\ 2)$$

31.12 3)

$$(\Sigma):\begin{cases}\log x+\log y=1\\ 9^{x-2y}\cdot 3^y=81\end{cases}$$

Περιορισμοί : i) $x>0$ ii) $y>0$

$$(\Sigma):\Rightarrow\begin{cases}\log xy=1\\ 3^{2x-4y}\cdot 3^y=81\end{cases}\Rightarrow\begin{cases}\log xy=\log 10\\ 3^{2x-3y}=3^4\end{cases}\Rightarrow\begin{cases}xy=10\\ 2x-3y=4\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}xy=10\\ x=\frac{4+3y}{2}\end{cases}\Rightarrow\begin{cases}\frac{(4+3y)y}{2}=10\\ x=\frac{4+3y}{2}\end{cases}\Rightarrow\begin{cases}3y^2+4y-20=0\\ x=\frac{4+3y}{2}\end{cases}\Rightarrow$$

$$\overset{\Delta=256}{\underset{y_{1,2}=\frac{-10}{3},2}{\Rightarrow}}\begin{cases}(3y+10)(y-2)=0\\ x=\frac{4+3y}{2}\end{cases}\overset{y>0}{\Rightarrow}\begin{cases}y=2\\ x=5\end{cases}$$ δεκτή

31.12 4)

$$(\Sigma):\begin{cases}3^{\log x}+5^{\log y}=14\\ 9^{\log x}-25^{\log y}=56\end{cases}$$

Περιορισμοί : i) $x>0$ ii) $y>0$

$$(\Sigma):\Rightarrow\begin{cases}3^{\log x}+5^{\log y}=14\\ 3^{2\log x}-5^{2\log y}=56\end{cases}\overset{\substack{3^{\log x}=\alpha>0\\ 5^{\log y}=\beta>0}}{\Rightarrow}\begin{cases}\alpha+\beta=14\\ \alpha^2-\beta^2=56\end{cases}\Rightarrow\begin{cases}\alpha+\beta=14\\ (\alpha+\beta)(\alpha-\beta)=56\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\alpha+\beta=14\\ 14(\alpha-\beta)=56\end{cases}\Rightarrow\begin{cases}\alpha+\beta=14\\ \alpha-\beta=4\end{cases}\Rightarrow\begin{cases}\alpha+\alpha-4=14\\ \beta=\alpha-4\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\alpha=9\\ \beta=5\end{cases}\Rightarrow\begin{cases}3^{\log x}=9\\ 5^{\log y}=5\end{cases}\Rightarrow\begin{cases}3^{\log x}=3^2\\ 5^{\log y}=5^1\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\log x=2\\ \log y=1\end{cases}\Rightarrow\begin{cases}x=100\\ y=10\end{cases}\Rightarrow(x\,,\,y)=(100\,,\,2)$$ δεκτή

31.12 5)

$$(\Sigma):\begin{cases}x^{\log y}=100\\ xy=1000\end{cases}$$

Περιορισμοί : $y>0$

$$(\Sigma):\Rightarrow\begin{cases}\log x^{\log y}=\log 100\\ \log xy=\log 1000\end{cases}\Rightarrow\begin{cases}\log y\cdot\log x=2\\ \log x+\log y=3\end{cases}\overset{\substack{\log x=\alpha\\ \log y=\beta}}{\Rightarrow}\begin{cases}\alpha\beta=2\\ \alpha+\beta=3\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\alpha(3-\alpha)=2\\ \beta=3-\alpha\end{cases}\Rightarrow\begin{cases}\alpha^2-3\alpha+2=0\\ \beta=3-\alpha\end{cases}\Rightarrow$$

$$\overset{\Delta=1}{\Rightarrow}{}_{\alpha_{1,2}=\frac{3\pm1}{2}\begin{matrix}/1\\ \backslash 2\end{matrix}}\begin{cases}\alpha=9 \text{ ή } \alpha=2\\ \beta=5 \text{ ή } \beta=1\end{cases}\Rightarrow\begin{cases}\log x=1 \text{ ή } \log x=2\\ \log y=2 \text{ ή } \log y=1\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}x=10 \text{ ή } x=100\\ y=100 \text{ ή } y=10\end{cases}\Rightarrow$$

$$\Rightarrow(x,\,y)=(10\,,100) \text{ ή } (x\,,\,y)=(100\,,10)$$ δεκτές

31.12 6)

$$(\Sigma):\begin{cases} x^{\log y}=\dfrac{1}{100} \\ 2^{\log y}\cdot 4^{\log x}=1 \end{cases}$$

$$\text{Περιορισμοί}:\begin{cases} y>0 \\ x>0 \end{cases}$$

$$(\Sigma):\Rightarrow\begin{cases} \log x^{\log y}=\log\dfrac{1}{100} \\ 2^{2\log y}\cdot 2^{2\log x}=1 \end{cases}\Rightarrow\begin{cases} \log x\cdot\log y=-2 \\ 2^{\log y+2\log x}=2^0 \end{cases}\Rightarrow\begin{cases} \log x\cdot\log y=-2 \\ \log y+2\log x=0 \end{cases}\Rightarrow$$

$$\overset{\substack{\alpha=\log x\\ \beta=\log y}}{\Rightarrow}\begin{cases} \alpha\beta=-2 \\ \beta+2\alpha=0 \end{cases}\Rightarrow\begin{cases} 2\alpha^2=2 \\ \beta=-2\alpha \end{cases}\Rightarrow\begin{cases} \alpha=1 \quad \text{ή} \quad \alpha=-1 \\ \beta=-1 \quad \text{ή} \quad \beta=2 \end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases} \log x=1 \quad \text{ή} \quad \log x=x-1 \\ \log y=-2 \quad \text{ή} \quad \log y=2 \end{cases}\Rightarrow\begin{cases} x=10 \quad \text{ή} \quad x=\dfrac{1}{10} \\ y=\dfrac{1}{100} \quad \text{ή} \quad y=100 \end{cases}\Rightarrow$$

$$\Rightarrow (x\ ,\ y)=\left(10\ ,\ \frac{1}{100}\right)\quad \text{ή}\quad (x\ ,\ y)=\left(\frac{1}{10}\ ,\ 100\right)\qquad \text{δεκτές}$$

31.12 7)

$$(\Sigma):\begin{cases} \log x^3-2\log y=7 \\ \log x-\log y^5=-2 \end{cases}$$

$$\text{Περιορισμοί}:\begin{cases} x>0 \\ y>0 \end{cases}$$

$$(\Sigma):\Rightarrow\begin{cases} 3\log x-2\log y=7 \\ \log x-5\log y=-2 \end{cases}\overset{\substack{\log x=\alpha\\ \log y=\beta}}{\Rightarrow}\begin{cases} 3\alpha-2\beta=7 \\ \alpha-5\beta=-2 \end{cases}\Rightarrow\begin{cases} 3\alpha-2\beta=7 \\ \alpha=-2+5\beta \end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases} -6+15\beta-2\beta=7 \\ \alpha=-2+5\beta \end{cases}\Rightarrow\begin{cases} 13\beta=13 \\ \alpha=-2+5\beta \end{cases}\Rightarrow\begin{cases} \beta=1 \\ \alpha=3 \end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases} \log y=1 \\ \log x=3 \end{cases}\Rightarrow\begin{cases} y=10 \\ x=1000 \end{cases}\Rightarrow (x\ ,\ y)=(1000\ ,\ 10)\qquad \text{δεκτή}$$

31.12 8)

$$(\Sigma):\begin{cases} \log(20x+10y)=2 \\ \log(x+1)+\log y=1 \end{cases}$$

$$\text{Περιορισμοί}:\begin{cases} y>0 \\ x+1>0\Rightarrow x>-1 \\ 20x+10y>0\Rightarrow 2x+y>0 \end{cases}$$

$$(\Sigma):\Rightarrow\begin{cases}\log(20x+10y)=\log 100\\ \log[(x+1)y]=\log 10\end{cases}\Rightarrow\begin{cases}20x+10y=100\\ (x+1)y=10\end{cases}\Rightarrow\begin{cases}2x+y=10\\ (x+1)y=10\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}y=10-2x\\ (x+1)(10-2x)=10\end{cases}\Rightarrow\begin{cases}y=10-2x\\ 10x-2x^2+10-2x=10\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}y=10-2x\\ 2x^2-8x=0\end{cases}\Rightarrow\begin{cases}y=10-2x\\ 2x(x-4)=0\end{cases}\Rightarrow\begin{cases}y=10 & \text{ή} & y=2\\ x=0 & \text{ή} & x=4\end{cases}\Rightarrow$$

$$\Rightarrow (x\ ,\ y)=(0\ ,\ 10)\quad \text{ή}\quad (x\ ,\ y)=(4\ ,\ 2)\qquad\qquad \text{δεκτές}$$

31.12 9)

$$(\Sigma):\begin{cases}\log x^2-\log y^4=-10\\ \log^2 x+\log^2 y=5\end{cases}$$

Περιορισμοί : $\begin{cases}x>0\\ y>0\end{cases}$

$$(\Sigma):\Rightarrow\begin{cases}2\log x-4\log y=-10\\ \log^2 x+\log^2 y=5\end{cases}\overset{\substack{\log x=\alpha\\ \log y=\beta}}{\Rightarrow}\begin{cases}2\alpha-4\beta=-10\\ \alpha^2+\beta^2=5\end{cases}\Rightarrow\begin{cases}\alpha-2\beta=-5\\ \alpha^2+\beta^2=5\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\alpha=2\beta-5\\ (2\beta-5)^2+\beta^2=5\end{cases}\Rightarrow\begin{cases}\alpha=2\beta-5\\ 4\beta^2-20\beta+25+\beta^2=5\end{cases}\Rightarrow\begin{cases}\alpha=2\beta-5\\ 5\beta^2-20\beta+20=0\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\alpha=2\beta-5\\ \beta^2-4\beta+4=0\end{cases}\Rightarrow\begin{cases}\alpha=2\beta-5\\ (\beta-2)^2=0\end{cases}\Rightarrow\begin{cases}\alpha=-1\\ \beta=2\end{cases}\Rightarrow\begin{cases}\log x=-1\\ \log y=2\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}x=\dfrac{1}{10}\\ y=100\end{cases}\Rightarrow (x\ ,\ y)=\left(\frac{1}{10}\ ,\ 100\right)\qquad\qquad \text{δεκτή}$$

31.12 10)

$$(\Sigma):\Rightarrow\begin{cases}3^x-2^y=1\\ x\ln 3+y\ln 2=\ln 72\end{cases}\Rightarrow\begin{cases}3^x-2^y=1\\ \ln 3^x+\ln 2^y=\ln 72\end{cases}\Rightarrow\begin{cases}3^x-2^y=1\\ \ln 3^x2^y=\ln 72\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}3^x-2^y=1\\ 3^x2^y=72\end{cases}\Rightarrow\begin{cases}3^x=1+2^y\\ 2^y(1+2^y)=72\end{cases}\Rightarrow\begin{cases}3^x=1+2^y\\ 2^{2y}+2^y-72=0\end{cases}\Rightarrow$$

$$\overset{2^y=\omega}{\Rightarrow}\begin{cases}3^x=1+2^y\\ \omega^2+\omega-72=0\end{cases}\overset{\substack{\Delta=289\\ \omega_{1,2}=\frac{-1\pm 17}{2}\left\langle\begin{matrix}8\\ -9\end{matrix}\right.}}{\Rightarrow}\begin{cases}3^x=1+2^y\\ \omega=8\ \text{ή}\ \omega=-9\end{cases}\Rightarrow$$

$$\Rightarrow \begin{cases} 3^x = 1 + 2^y \\ 2^y = 8 \ \text{ή} \ 2^y = -9 < 0 \quad (\text{αδύνατη}) \end{cases} \Rightarrow \begin{cases} 3^x = 1 + 8 \\ y = 3 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 3^x = 9 \\ y = 3 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 3 \end{cases}$$