

29.8 1)

Καταρχήν πρέπει $\begin{cases} x > 0 \\ x+9 > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x > -9 \end{cases} \xrightarrow{\text{συναλήθευση}} x > 0$. Με τον περιορισμό αυτό έχουμε

$$1 + \frac{1}{2} \log x = \log(x+9) \Rightarrow \log 10 + \log x^{\frac{1}{2}} = \log(x+9) \Rightarrow \log 10\sqrt{x} = \log(x+9) \Rightarrow 10\sqrt{x} = x+9 \Rightarrow$$

$$\begin{aligned} \text{θέτουμε } y = \sqrt{x} \\ \Rightarrow 10y = y^2 + 9 \Rightarrow y^2 - 10y + 9 = 0 \Rightarrow y = 1 \text{ ή } y = 9 \Rightarrow \sqrt{x} = 1 \text{ ή } \sqrt{x} = 9 \Rightarrow x = 1 \text{ ή } x = 81 \end{aligned}$$

οι οποίες γίνονται **δεκτές** καθώς ικανοποιούν τον περιορισμό

29.8 2)

$$\frac{1}{2} \log x - \log 4 = \log(x+1) - 1$$

περιορισμοί: $\begin{cases} x > 0 \\ x+1 > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x > -1 \end{cases} \xrightarrow{\text{συναλήθευση}} x > 0$

$$\log \sqrt{x} - \log 4 = \log(x+1) - \log 10 \Rightarrow$$

$$\Rightarrow \log \left(\frac{\sqrt{x}}{4} \right) = \log \left(\frac{x+1}{10} \right) \Rightarrow \frac{\sqrt{x}}{4} = \frac{x+1}{10} \Rightarrow$$

$$\Rightarrow 10\sqrt{x} = 4x + 4 \Rightarrow 4x - 10\sqrt{x} + 4 = 0 \Rightarrow 2x - 5\sqrt{x} + 2 = 0 \Rightarrow$$

$$\begin{aligned} \sqrt{x} = \omega \\ \Rightarrow 2\omega^2 - 5\omega + 2 = 0 \end{aligned}$$

$$\Delta = 9 \quad \omega_{1,2} = \frac{5 \pm 3}{4} \begin{cases} \nearrow x_1 = 2 \\ \searrow x_2 = \frac{1}{2} \end{cases}$$

Άρα: $* \omega = 2 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$ δεκτή

$* \omega = \frac{1}{2} \Rightarrow \sqrt{x} = \frac{1}{2} \Rightarrow x = \frac{1}{4}$ δεκτή

29.8 3)

$$\frac{1}{2} \log(x+24) = 1 - \log \sqrt{x+3}$$

περιορισμοί: $\begin{cases} * x+24 > 0 \Rightarrow x > -24 \\ x+3 > 0 \Rightarrow x > -3 \end{cases} \xrightarrow{\text{συναλήθευση}} x > -3$

$$\log \sqrt{x+24} = \log 10 - \log \sqrt{x+3} \Rightarrow$$

$$\Rightarrow \log \sqrt{x+24} = \log \left(\frac{10}{\sqrt{x+3}} \right) \Rightarrow \sqrt{x+24} = \frac{10}{\sqrt{x+3}} \Rightarrow$$

$$\Rightarrow \sqrt{(x+24)(x+3)} = 10 \Rightarrow \sqrt{x^2 + 3x + 24x + 72} = 10 \Rightarrow$$

$$\Rightarrow x^2 + 27x + 72 = 100 \Rightarrow x^2 + 27x - 28 = 0$$

$$\Delta = 841$$

$$x_{1,2} = \frac{-27 \pm 29}{2} \begin{array}{l} \nearrow x_1 = 1 \quad \text{δεκτή} \\ \searrow x_2 = -28 \quad \text{απορρίπτεται} \end{array}$$