29.18 1)

$$f(\alpha)=f(\beta) \Rightarrow \ln(\alpha+1)-\alpha=\ln(\beta+1)-\beta \Rightarrow \ln(\alpha+1)-\ln e^{\alpha}=\ln(\beta+1)-\ln e^{\beta} \Rightarrow$$

$$\Rightarrow \ln\left(\frac{\alpha+1}{e^{\alpha}}\right)=\ln\left(\frac{\beta+1}{e^{\beta}}\right) \Rightarrow \frac{\alpha+1}{e^{\alpha}}=\frac{\beta+1}{e^{\beta}} \Rightarrow \frac{e^{\alpha}}{e^{\beta}}=\frac{\alpha+1}{\beta+1} \Rightarrow e^{\alpha-\beta}=\frac{\alpha+1}{\beta+1}$$

29.18 2)

$$f(\alpha)=f(\beta) \Rightarrow \frac{\ln\alpha}{\alpha}=\frac{\ln\beta}{\beta} \Rightarrow \beta\ln\alpha=\alpha\ln\beta \Rightarrow \ln\alpha^{\beta}=\ln\beta^{\alpha} \Rightarrow \alpha^{\beta}=\beta^{\alpha}$$

29.18 3)

$$f(x)=\frac{\ln(x+1)}{x} \quad , \ \alpha \ , \ \beta>0$$

$$f(\alpha)=f(\beta) \Rightarrow \frac{\ln(\alpha+1)}{\alpha}=\frac{\ln(\beta+1)}{\beta} \Rightarrow \beta\ln(\alpha+1)=\alpha\ln(\beta+1) \Rightarrow$$

$$\Rightarrow \ln(\alpha+1)^{\beta}=\ln(\beta+1)^{\alpha} \Rightarrow (\alpha+1)^{\beta}=(\beta+1)^{\alpha}$$

29.18 4)

$$f(x)=\frac{\ln x}{x-1} \quad , \ \alpha \ , \ \beta>0$$

$$f(\alpha)=f(\beta) \Rightarrow \frac{\ln\alpha}{\alpha-1}=\frac{\ln\beta}{\beta-1} \Rightarrow \beta\ln\alpha-\ln\alpha=\alpha\ln\beta-\ln\beta \Rightarrow$$

$$\Rightarrow \ln\alpha^{\beta}-\ln\alpha=\ln\beta^{\alpha}-\ln\beta \Rightarrow \ln\frac{\alpha^{\beta}}{\alpha}=\ln\frac{\beta^{\alpha}}{\beta} \Rightarrow$$

$$\Rightarrow \frac{\alpha^{\beta}}{\alpha}=\frac{\beta^{\alpha}}{\beta} \Rightarrow \beta\alpha^{\beta}=\alpha\beta^{\alpha}$$

29.18 5)

$$f(x)=\frac{x\ln x}{x+1} \quad , \ \alpha \ , \ \beta>0$$

$$f(\alpha)=f(\beta) \Rightarrow \frac{\alpha\ln\alpha}{\alpha+1}=\frac{\beta\ln\beta}{\beta+1} \Rightarrow \alpha\beta\ln\alpha+\alpha\ln\alpha=\alpha\beta\ln\beta+\beta\ln\beta \Rightarrow$$

$$\Rightarrow \ln\alpha^{\alpha\beta}+\ln\alpha^{\alpha}=\ln\beta^{\alpha\beta}+\ln\beta^{\beta} \Rightarrow \ln\left(\alpha^{\alpha\beta}\cdot\alpha^{\alpha}\right)=\ln\left(\beta^{\alpha\beta}\cdot\beta^{\beta}\right) \Rightarrow$$

$$\Rightarrow \alpha^{\alpha\beta}\cdot\alpha^{\alpha}=\beta^{\alpha\beta}\cdot\beta^{\beta} \Rightarrow \frac{\alpha^{\alpha\beta}}{\beta^{\alpha\beta}}=\frac{\beta^{\beta}}{\alpha^{\alpha}}$$

29.18 6)

$$f(x)=x-\alpha\ln x \quad , \ \alpha>0$$

$$f(\alpha)=f(e)$$

$$\Rightarrow \alpha-\alpha\ln\alpha=e-\alpha\ln e \Rightarrow \ln e^{\alpha}-\ln\alpha^{\alpha}=\ln e^{e}-\ln e^{\alpha} \Rightarrow$$

$$\Rightarrow \ln\left(\frac{e^{\alpha}}{\alpha^{\alpha}}\right)=\ln\left(\frac{e^{e}}{e^{\alpha}}\right) \Rightarrow \frac{e^{\alpha}}{\alpha^{\alpha}}=\frac{e^{e}}{e^{\alpha}} \Rightarrow e^{2\alpha}=\alpha^{\alpha}\cdot e^{e}$$

29.18 7)

$$f(x)=\frac{\ln(x-1)}{\ln x}-x \quad , \ \alpha \ , \ \beta>1$$

$$f(\alpha)=f(\beta) \Rightarrow \frac{\ln(\alpha-1)}{\ln\alpha}=\frac{\ln(\beta-1)}{\ln\beta} \Rightarrow$$

$$\Rightarrow \ln\beta\cdot\ln(\alpha-1)=\ln\alpha\cdot\ln(\beta-1) \Rightarrow \ln\beta^{\ln(\alpha-1)}=\ln\alpha^{\ln(\beta-1)} \Rightarrow$$

$$\Rightarrow \beta^{\ln(\alpha-1)}=\alpha^{\ln(\beta-1)}$$

29.18 8)

$$f(x)=e^{x}(1-x)-1 \quad , \ \alpha \ , \ \beta<1$$

$$f(\alpha)=f(\beta) \Rightarrow e^{\alpha}(1-\alpha)-1=e^{\beta}(1-\beta)-1 \Rightarrow$$

$$\Rightarrow e^{\alpha}(1-\alpha)=e^{\beta}(1-\beta) \Rightarrow \frac{e^{\alpha}}{e^{\beta}}=\frac{1-\beta}{1-\alpha} \Rightarrow$$

$$\Rightarrow e^{\alpha-\beta}=\frac{1-\beta}{1-\alpha} \Rightarrow \ln e^{\alpha-\beta}=\ln\left(\frac{1-\beta}{1-\alpha}\right) \Rightarrow$$

$$\Rightarrow \alpha-\beta=\ln(1-\beta)-\ln(1-\alpha)$$

29.18 9)

$$f(x)=x^{2}e^{x} \quad , \ \alpha \ , \ \beta>0$$

$$f(\alpha)=f(\beta) \Rightarrow \alpha^{2}e^{\alpha}=\beta^{2}e^{\beta} \Rightarrow \frac{e^{\alpha}}{e^{\beta}}=\frac{\beta^{2}}{\alpha^{2}} \Rightarrow e^{\alpha-\beta}=\frac{\beta^{2}}{\alpha^{2}} \Rightarrow$$

$$\Rightarrow \ln e^{\alpha-\beta}=\ln\left(\frac{\beta^{2}}{\alpha^{2}}\right) \Rightarrow \alpha-\beta=\ln\beta^{2}-\ln\alpha^{2} \Rightarrow$$

$$\Rightarrow \alpha-\beta=2\ln\beta-2\ln\alpha$$

29.18 10)

$$f(x)=x^{2}-\ln x \quad , \ \alpha \ , \ \beta>0$$

$$f(\alpha)=f(\beta) \Rightarrow \alpha^{2}-\ln\alpha=\beta^{2}-\ln\beta \Rightarrow \ln e^{\alpha^{2}}-\ln\alpha=\ln e^{\beta^{2}}-\ln\beta \Rightarrow$$

$$\Rightarrow \ln\left(\frac{e^{\alpha^{2}}}{\alpha}\right)=\ln\left(\frac{e^{\beta^{2}}}{\beta}\right) \Rightarrow \frac{e^{\alpha^{2}}}{\alpha}=\frac{e^{\beta^{2}}}{\beta} \Rightarrow \beta e^{\alpha^{2}}=\alpha e^{\beta^{2}}$$