

29.11 1)

Καταρχήν πρέπει $x > 0$. Με τον περιορισμό αυτό "λογαριθμίζουμε" τα δύο μέλη και έχουμε

$$x^{\log x - 2} = 100x \Rightarrow \log x^{\log x - 2} = \log(100x) \Rightarrow (\log x - 2)\log x = \log(100x) \Rightarrow$$

$$\Rightarrow \log^2 x - 2\log x = \log 100 + \log x \xrightarrow{\log 100 = 2} \log^2 x - 2\log x + \log x - 2 = 0 \Rightarrow \log^2 x - \log x - 2 = 0 \Rightarrow$$

$$\begin{array}{l} \Delta = 1 + 8 = 9, \quad x_{1,2} = \frac{1 \pm \sqrt{9}}{2 \cdot 1} = \frac{1 \pm 3}{2} \Rightarrow \begin{array}{l} x_1 = \frac{1-3}{2} = -1 \\ x_2 = \frac{1+3}{2} = 2 \end{array} \\ \text{Θέτουμε } y = \log x \\ \Rightarrow y^2 - y - 2 = 0 \end{array}$$

$$\Rightarrow y = -1$$

$$\text{ή } y = 2 \Rightarrow$$

$$\Rightarrow \log x = -1 \Rightarrow \log x = \log 10^{-1} \Rightarrow x = \frac{1}{10} \quad \parallel \parallel \parallel \quad \Rightarrow \log x = 2 \Rightarrow \log x = \log 10^2 \Rightarrow x = 100$$

29.11 2)

Κατ' αρχήν πρέπει $x > 0$.

Με τον περιορισμό αυτό λογαριθμίζουμε και τα δύο μέλη και έχουμε :

$$x^{\log x} = 10 \Rightarrow \log(x^{\log x}) = \log 10 \Rightarrow \log x \cdot \log x = 1 \Rightarrow$$

$$\Rightarrow \log^2 x = 1 \xrightarrow{y = \log x} y^2 = 1 \Rightarrow$$

$$\Rightarrow y = 1$$

$$\Rightarrow y = -1$$

$$\Rightarrow \log x = 1$$

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$$\Rightarrow \log x = -1$$

$$\Rightarrow x = 10$$

$$\Rightarrow x = 10^{-1}$$

29.11 3)

Περιορισμοί : $x > 0$

$$x^{\log x - 1} = 100 \Rightarrow \log x^{\log x - 1} = \log 100 \Rightarrow (\log x - 1)\log x = 2 \Rightarrow (\log x)^2 - \log x - 2 = 0$$

Θέτω $\log x = \omega$ και η εξίσωση γράφεται :

$$\omega^2 - \omega - 2 = 0$$

$$\Delta = 9 \quad \omega_{1,2} = \frac{1 \pm 3}{2} \Rightarrow \begin{array}{l} \omega_1 = 2 \\ \omega_2 = -1 \end{array}$$

Άρα : i) $\omega = 2 \Rightarrow \log x = 2 \Rightarrow x = 100$ δεκτή

ii) $\omega = -1 \Rightarrow \log x = -1 \Rightarrow x = \frac{1}{10}$ δεκτή

29.11 4)

Κατ' αρχήν πρέπει $x > 0$.

Με τον περιορισμό αυτό λογαριθμίζουμε και τα δύο μέλη και έχουμε :

$$\begin{aligned}
x^{\frac{1+\log x}{2}} &= x \Rightarrow \log x^{\frac{1+\log x}{2}} = \log x \Rightarrow \left(\frac{1+\log x}{2}\right) \log x = \log x \Rightarrow \\
\Rightarrow \log x \left(\frac{1+\log x}{2} - 1\right) &= 0 \Rightarrow \log x \cdot \frac{\log x - 1}{2} = 0 \Rightarrow \\
\stackrel{y=\log x}{\Rightarrow} y \cdot \frac{y-1}{2} &= 0 \Rightarrow y=0 \quad \text{ή} \quad \frac{y-1}{2} = 0 \Rightarrow \\
\Rightarrow y=0 & \qquad \qquad \qquad \Rightarrow y=1 \\
\Rightarrow \log x=0 & \qquad \qquad \text{ή} \qquad \qquad \Rightarrow \log x=1 \\
\Rightarrow x=1 & \qquad \qquad \qquad \Rightarrow x=10
\end{aligned}$$

29.11 5)

Κατ' αρχήν πρέπει $x > 0$.

Με τον περιορισμό αυτό λογαριθμίζουμε και τα δύο μέλη και έχουμε :

$$\begin{aligned}
x^{\frac{\log x - 1}{2}} &= 10 \Rightarrow \log x^{\frac{\log x - 1}{2}} = \log 10 \Rightarrow \frac{\log x - 1}{2} \cdot \log x = 1 \Rightarrow \log x \left(\frac{\log x - 1}{2}\right)^{-1} = 0 \Rightarrow \\
\stackrel{y=\log x}{\Rightarrow} y \left(\frac{4-1}{2}\right)^{-1} &= 0 \Rightarrow y^2 - y - 2 = 0 \Rightarrow \\
\Rightarrow y = -1 & \qquad \qquad \qquad \Rightarrow y = 2 \\
\Rightarrow \log x = -1 & \qquad \qquad \text{ή} \qquad \qquad \Rightarrow \log x = 2 \\
\Rightarrow x = 10^{-1} & \qquad \qquad \qquad \Rightarrow x = 100
\end{aligned}$$

29.11 6)

Περιορισμοί : $x > 0$

$$\begin{aligned}
x^{\frac{\log x + 5}{3}} &= 10^{5+\log x} \Rightarrow \log x^{\frac{\log x + 5}{3}} = \log 10^{5+\log x} \Rightarrow \log x \frac{(\log x + 5)}{3} = \log x + 5 \Rightarrow \\
\Rightarrow (\log x + 5) \left(\frac{\log x}{3} - 1\right) &= 0 \Rightarrow \\
\Rightarrow \log x = -5 & \qquad \text{ή} \qquad \log x = 3 \\
\Rightarrow x = 10^{-5} = \frac{1}{100000} & \qquad \text{ή} \qquad x = 1000
\end{aligned}$$

29.11 7)

Κατ' αρχήν πρέπει $x > 0$.

Με τον περιορισμό αυτό λογαριθμίζουμε και τα δύο μέλη και έχουμε :

$$\begin{aligned}
x^{\log x + 1} &= 10x \Rightarrow \log x^{\log x + 1} = \log 10x \Rightarrow (\log x + 1) \log x = \log 10 + \log x \Rightarrow \\
\stackrel{\log x = y}{\Rightarrow} (y+1)y &= 1+y \Rightarrow (y+1)(y-1) = 0 \Rightarrow \\
\Rightarrow y = -1 & \qquad \qquad \qquad \Rightarrow y = 1 \\
\Rightarrow \log x = -1 & \qquad \text{ή} \qquad \qquad \Rightarrow \log x = 1 \\
\Rightarrow x = 10^{-1} & \qquad \qquad \qquad \Rightarrow x = 10
\end{aligned}$$

29.11 8)

Κατ' αρχήν πρέπει $x > 1$.

Με τον περιορισμό αυτό λογαριθμίζουμε και τα δύο μέλη και έχουμε :

$$\begin{aligned}
(x-1)^{\log(x-1)} &= 10^{\log(x-1)} \Rightarrow \log(x-1)^{\log(x-1)} = \log 10^{\log(x-1)} \Rightarrow \\
\Rightarrow \log(x-1) \log(x-1) &= \log(x-1) \cdot \log 10 \Rightarrow \\
\Rightarrow \log^2(x-1) &= \log(x-1) \Rightarrow \log(x-1)(\log(x-1) - 1) = 0 \Rightarrow
\end{aligned}$$

$$y = \log(x-1) \\ \Rightarrow y(y-1) = 0 \Rightarrow$$

$$\Rightarrow y = 0$$

$$\Rightarrow \log(x-1) = 0$$

$$\Rightarrow x-1 = 0$$

$$\Rightarrow x = 1$$

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$$\Rightarrow y = 1$$

$$\Rightarrow \log(x-1) = 1$$

$$\Rightarrow x-1 = 10$$

$$\Rightarrow x = 11$$