28.4 1)

α) Έστω $\log_{\frac{1}{2}} 32 = x$. Τότε

$\log_{\frac{1}{2}} 32 = x \Leftrightarrow \left(\frac{1}{2}\right)^x = 32 \Leftrightarrow 2^{-x} = 2^5 \Leftrightarrow -x = 5 \Leftrightarrow x = -5$. Επομένως $\log_{\frac{1}{2}} 32 = -5$

β) Έστω $\log_{27} \frac{1}{9\sqrt{3}} = x$. Τότε $\log_{27} \frac{1}{9\sqrt{3}} = x \Leftrightarrow 27^x = 9\sqrt{3} \Leftrightarrow$

$\Leftrightarrow \left(3^3\right)^x = 3^2 \cdot 3^{\frac{1}{2}} \Leftrightarrow 3^{3x} = 3^{2+\frac{1}{2}} \Leftrightarrow 3x = \frac{5}{2} \Leftrightarrow x = \frac{5}{6}$. $\log_{27} \frac{1}{9\sqrt{3}} = \frac{5}{6}$

γ) Έστω $\log_{\sqrt{\frac{5}{4}}} \frac{16}{25} = x$. Τότε $\log_{\sqrt{\frac{5}{4}}} \frac{16}{25} = x \Leftrightarrow \sqrt{\frac{5}{4}}^x = \frac{16}{25} \Leftrightarrow \left(\frac{5}{4}\right)^{\frac{x}{2}} = \left(\frac{4}{5}\right)^2 \Leftrightarrow$

$\Leftrightarrow \left(\frac{5}{4}\right)^{\frac{x}{2}} = \left(\frac{5}{4}\right)^{-2} \Leftrightarrow \frac{x}{2} = -2 \Leftrightarrow x = -4$. Επομένως $\log_{\sqrt{\frac{5}{4}}} \frac{16}{25} = -4$

28.4 2)

$\log_{\frac{1}{2}} 64 = x \Rightarrow \left(\frac{1}{2}\right)^x = 64 \Rightarrow 2^{-x} = 2^6 \Rightarrow -x = 6 \Rightarrow x = -6$

28.4 3)

$\log_{\frac{1}{25}} 125 = x \Rightarrow \left(\frac{1}{25}\right)^x = 125 \Rightarrow 25^{-x} = 125 \Rightarrow 5^{-2x} = 5^3 \Rightarrow$

$\Rightarrow -2x = 3 \Rightarrow x = -\frac{3}{2}$

28.4 4)

$\log_9 \sqrt{3} = x \Rightarrow 9^x = \sqrt{3} \Rightarrow 3^{2x} = 3^{\frac{1}{2}} \Rightarrow 2x = \frac{1}{2} \Rightarrow x = \frac{1}{4}$

28.4 5)

$\log_{\sqrt{8}} \sqrt{2} = x \Rightarrow \left(\sqrt{8}\right)^x = \sqrt{2} \Rightarrow 2^{\frac{3x}{2}} = 2^{\frac{1}{2}} \Rightarrow \frac{3x}{2} = \frac{1}{2} \Rightarrow x = \frac{1}{3}$

28.4 6)

$\log_{25} \sqrt{5} = x \Rightarrow 25^x = \sqrt{5} \Rightarrow 5^{2x} = 5^{\frac{1}{2}} \Rightarrow 2x = \frac{1}{2} \Rightarrow x = \frac{1}{4}$

28.4 7)

$\log_{\sqrt{5}} 125 = x \Rightarrow \left(\sqrt{5}\right)^x = 125 \Rightarrow 5^{\frac{x}{2}} = 5^3 \Rightarrow \frac{x}{2} = 3 \Rightarrow x = 6$

28.4 8)

$\log_{\sqrt{2}} 64 = x \Rightarrow \left(\sqrt{2}\right)^x = 64 \Rightarrow 2^{\frac{x}{2}} = 2^6 \Rightarrow \frac{x}{2} = 6 \Rightarrow x = 12$

28.4 9)

$\log_{16} \frac{1}{2\sqrt{2}} = x \Rightarrow (16)^x = \left(2\sqrt{2}\right)^{-1} \Rightarrow 2^{4x} = \left(2 \cdot 2^{\frac{1}{2}}\right)^{-1} \Rightarrow$

$$\Rightarrow 2^{4x} = 2^{-\frac{3}{2}} \Rightarrow 4x = -\frac{3}{2} \Rightarrow x = -\frac{3}{8}$$

28.4 10)

Έστω $\log_{\sqrt{\frac{2}{3}}} \frac{81}{16} = x$. Τότε :

$$\log_{\sqrt{\frac{2}{3}}} \frac{81}{16} = x \Rightarrow \sqrt{\frac{2}{3}}^{x} = \frac{81}{16} \Rightarrow \left(\frac{2}{3}\right)^{\frac{x}{2}} = \frac{3^4}{2^4} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x}{2}} = \left(\frac{2}{3}\right)^{-4} \Rightarrow \frac{x}{2} = -4 \Rightarrow x = -8$$

28.4 11)

Έστω $\log_{\frac{9}{25}} \sqrt[7]{\frac{125}{27}} = x$. Τότε :

$$\log_{\frac{9}{25}} \sqrt[7]{\frac{125}{27}} = x \Rightarrow \left(\frac{9}{25}\right)^{x} = \sqrt[7]{\frac{125}{27}} \Rightarrow \left(\frac{3^2}{5^2}\right)^{x} = \sqrt[7]{\frac{5^3}{3^3}} \Rightarrow$$

$$\Rightarrow \left(\frac{3}{5}\right)^{2x} = \left(\frac{3}{5}\right)^{-\frac{3}{7}} \Rightarrow 2x = -\frac{3}{7} \Rightarrow x = -\frac{3}{14}$$

28.4 12)

$\log_{\sqrt{\alpha}} \alpha^3 = x \qquad (0 < \alpha \neq 1)$

$$\Rightarrow \left(\sqrt{\alpha}\right)^{x} = \alpha^3 \Rightarrow \alpha^{\frac{x}{2}} = \alpha^3 \Rightarrow \frac{x}{2} = 3 \Rightarrow x = 6$$