

28.20 1)

$$\begin{aligned} \beta' \text{ μέλος} &= \frac{\log_{\alpha} \theta \cdot \log_{\beta} \theta}{\log_{\alpha} \theta + \log_{\beta} \theta} = \frac{\frac{\log_{\alpha\beta} \theta}{\log_{\alpha\beta} \alpha} \cdot \frac{\log_{\alpha\beta} \theta}{\log_{\alpha\beta} \beta}}{\frac{\log_{\alpha\beta} \theta}{\log_{\alpha\beta} \alpha} + \frac{\log_{\alpha\beta} \theta}{\log_{\alpha\beta} \beta}} = \frac{\frac{\log_{\alpha\beta}^2 \theta}{\log_{\alpha\beta} \alpha \cdot \log_{\alpha\beta} \beta}}{\log_{\alpha\beta} \theta \left(\frac{1}{\log_{\alpha\beta} \alpha} + \frac{1}{\log_{\alpha\beta} \beta} \right)} = \\ &= \frac{\frac{\log_{\alpha\beta}^2 \theta}{\cancel{\log_{\alpha\beta} \alpha} \cdot \log_{\alpha\beta} \beta}}{\cancel{\log_{\alpha\beta} \theta} \frac{\log_{\alpha\beta} \beta + \log_{\alpha\beta} \alpha}{\cancel{\log_{\alpha\beta} \alpha} \cdot \log_{\alpha\beta} \beta}} = \frac{\log_{\alpha\beta} \theta}{\log_{\alpha\beta} \beta + \log_{\alpha\beta} \alpha} = \frac{\log_{\alpha\beta} \theta}{\log_{\alpha\beta} \alpha\beta} = \frac{\log_{\alpha\beta} \theta}{1} = \log_{\alpha\beta} \theta = \alpha' \text{ μέλος} \end{aligned}$$

28.20 2)

$$\log_{\alpha^6} \beta^4 \cdot \log_{\beta^2} \alpha^3 = \frac{\log_{\beta^2} \beta^4}{\log_{\beta^2} \alpha^6} \cdot \log_{\beta^2} \alpha^3 = \frac{2 \cdot \cancel{\log_{\beta^2} \beta^2}}{2 \log_{\beta^2} \alpha^3} \cdot \log_{\beta^2} \alpha^3 = 1$$

28.20 3)

$$\begin{aligned} \frac{1}{\log_{\alpha} \beta^2} + \frac{1}{\log_{\alpha} \beta^3} + \frac{1}{\log_{\alpha} \beta^6} &= \frac{1}{2 \log_{\alpha} \beta} + \frac{1}{3 \log_{\alpha} \beta} + \frac{1}{6 \log_{\alpha} \beta} = \frac{\frac{1}{2 \log_{\beta} \beta}}{\log_{\beta} \alpha} + \frac{\frac{1}{3 \log_{\beta} \beta}}{\log_{\beta} \alpha} + \frac{\frac{1}{6 \log_{\beta} \beta}}{\log_{\beta} \alpha} = \\ &= \frac{\log_{\beta} \alpha}{2} + \frac{\log_{\beta} \alpha}{3} + \frac{\log_{\beta} \alpha}{6} = \log_{\beta} \alpha \end{aligned}$$

28.20 4)

$$\frac{\log_{\beta} \alpha}{1 + \log_{\beta} \alpha} = \frac{\frac{\log_{\alpha\beta} \alpha}{\log_{\alpha\beta} \beta}}{1 + \frac{\log_{\alpha\beta} \alpha}{\log_{\alpha\beta} \beta}} = \frac{\log_{\alpha\beta} \alpha}{\log_{\alpha\beta} \beta + \log_{\alpha\beta} \alpha} = \frac{\log_{\alpha\beta} \alpha}{\log_{\alpha\beta} \alpha\beta} = \log_{\alpha\beta} \alpha$$

28.20 5)

$$\log_{\alpha} \beta^2 \cdot \log_{\alpha^2} \beta = 2 \log_{\alpha} \beta \cdot \log_{\alpha^2} \beta = 2 \log_{\alpha} \beta \cdot \frac{\log_{\alpha} \beta}{\log_{\alpha} \alpha^2} = \frac{2 (\log_{\alpha} \beta)^2}{2} = (\log_{\alpha} \beta)^2$$

28.20 6)

$$\begin{aligned} \log_{\alpha} \theta \cdot \log_{\beta} \theta + \log_{\beta} \theta \cdot \log_{\gamma} \theta + \log_{\gamma} \theta \cdot \log_{\alpha} \theta &= \frac{\log \theta}{\log \alpha} \cdot \frac{\log \theta}{\log \beta} + \frac{\log \theta}{\log \beta} \cdot \frac{\log \theta}{\log \gamma} + \frac{\log \theta}{\log \gamma} \cdot \frac{\log \theta}{\log \alpha} = \\ &= (\log \theta)^2 \left(\frac{1}{\log \alpha \log \beta} + \frac{1}{\log \beta \log \gamma} + \frac{1}{\log \alpha \log \gamma} \right) = (\log \theta)^2 \left(\frac{\log \alpha + \log \beta + \log \gamma}{\log \alpha \cdot \log \beta \cdot \log \gamma} \right) = \\ &= \frac{(\log \theta) \cdot (\log \theta) \cdot \log(\alpha\beta\gamma)}{\log \alpha \cdot \log \beta \cdot \log \gamma} = \frac{\log \theta}{\log \alpha} \cdot \frac{\log \theta}{\log \beta} \cdot \frac{\log(\alpha\beta\gamma)}{\log \gamma} = \end{aligned}$$

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βάσης

$$= \log_{\alpha} \theta \cdot \log_{\beta} \theta \cdot \frac{\log(\alpha\beta\gamma)}{\log \theta} \cdot \frac{\log \theta}{\log \gamma} = \log_{\alpha} \theta \cdot \log_{\beta} \theta \cdot \frac{1}{\log_{\alpha\beta\gamma} \theta} \cdot \log_{\gamma} \theta =$$

$$= \frac{\log_{\alpha} \theta \cdot \log_{\beta} \theta \cdot \log_{\gamma} \theta}{\log_{\alpha\beta\gamma} \theta}$$

28.20 7)

$$\text{i) } \log_{\alpha} \beta = \frac{\log_{\beta} \beta}{\log_{\beta} \alpha} = \frac{1}{\log_{\beta} \alpha}$$

$$\text{ii) } \alpha^{\frac{2}{\log_{\beta} \alpha}} - 2\alpha^{\frac{1}{\log_{\beta} \alpha}} \cdot \beta^{\frac{1}{\log_{\alpha} \beta}} + \beta^{\frac{2}{\log_{\alpha} \beta}}$$

$$= \alpha^{2\log_{\alpha} \beta} - 2\alpha^{\log_{\alpha} \beta} \cdot \beta^{\log_{\beta} \alpha} + \beta^{2\log_{\beta} \alpha} =$$

$$= \alpha^{\log_{\alpha} \beta^2} - 2 \cdot \beta \cdot \alpha + \beta^{\log_{\beta} \alpha^2} =$$

$$= \beta^2 - 2\alpha\beta + \alpha^2 = (\alpha - \beta)^2$$

28.20 8)

$$\frac{\log_{\alpha} \theta - \log_{\sqrt{\alpha\beta}} \theta}{\log_{\sqrt{\alpha\beta}} \theta - \log_{\beta} \theta}$$

$$\text{Είναι : } \log_{\sqrt{\alpha\beta}} \theta = \frac{\log_{\alpha\beta} \theta}{\log_{\alpha\beta} \sqrt{\alpha\beta}} = 2 \log_{\alpha\beta} \theta$$

$$\text{Άρα : } \frac{\log_{\alpha} \theta - \log_{\sqrt{\alpha\beta}} \theta}{\log_{\sqrt{\alpha\beta}} \theta - \log_{\beta} \theta} = \frac{\log_{\alpha} \theta - 2 \log_{\alpha\beta} \theta}{2 \log_{\alpha\beta} \theta - \log_{\beta} \theta} =$$

$$\stackrel{(1)}{=} \frac{\log_{\alpha} \theta - \frac{2 \log_{\alpha} \theta \log_{\beta} \theta}{\log_{\alpha} \theta + \log_{\beta} \theta}}{\frac{2 \log_{\alpha} \theta \log_{\beta} \theta}{\log_{\alpha} \theta + \log_{\beta} \theta} - \log_{\beta} \theta} = \frac{(\log_{\alpha} \theta)^2 - \log_{\alpha} \theta \log_{\beta} \theta}{\log_{\alpha} \theta \log_{\beta} \theta - (\log_{\beta} \theta)^2} = \frac{\log_{\alpha} \theta (\log_{\alpha} \theta - \log_{\beta} \theta)}{\log_{\beta} \theta (\log_{\alpha} \theta - \log_{\beta} \theta)} = \frac{\log_{\alpha} \theta}{\log_{\beta} \theta}$$