

Β ΛΥΚΕΙΟΥ ΑΛΓΕΒΡΑ

28.20 1)

$$\beta' \text{ μέλος} = \frac{\log_a \theta \cdot \log_\beta \theta}{\log_a \theta + \log_\beta \theta} = \frac{\frac{\log_{a\beta} \theta}{\log_{a\beta} \alpha} \cdot \frac{\log_{a\beta} \theta}{\log_{a\beta} \beta}}{\frac{\log_{a\beta} \theta}{\log_{a\beta} \alpha} + \frac{\log_{a\beta} \theta}{\log_{a\beta} \beta}} = \frac{\frac{\log^2_{a\beta} \theta}{\log_{a\beta} \alpha \cdot \log_{a\beta} \beta}}{\log_{a\beta} \theta \left(\frac{1}{\log_{a\beta} \alpha} + \frac{1}{\log_{a\beta} \beta} \right)} =$$

$$= \frac{\frac{\log^2_{a\beta} \theta}{\log_{a\beta} \alpha \cdot \log_{a\beta} \beta}}{\cancel{\log_{a\beta} \theta} \frac{\log_{a\beta} \beta + \log_{a\beta} \alpha}{\cancel{\log_{a\beta} \alpha \cdot \log_{a\beta} \beta}}} = \frac{\log_{a\beta} \theta}{\log_{a\beta} \beta + \log_{a\beta} \alpha} = \frac{\log_{a\beta} \theta}{\log_{a\beta} a\beta} = \frac{\log_{a\beta} \theta}{1} = \log_{a\beta} \theta = a' \text{ μέλος}$$

28.20 2)

$$\log_{a^6} \beta^4 \cdot \log_{\beta^2} a^3 = \frac{\log_{\beta^2} \beta^4}{\log_{\beta^2} a^6} \cdot \log_{\beta^2} a^3 = \frac{2 \cdot \cancel{\log_{\beta^2} \beta^2}^1}{2 \log_{\beta^2} a^3} \cdot \log_{\beta^2} a^3 = 1$$

28.20 3)

$$\begin{aligned} \frac{1}{\log_a \beta^2} + \frac{1}{\log_a \beta^3} + \frac{1}{\log_a \beta^6} &= \frac{1}{2 \log_a \beta} + \frac{1}{3 \log_a \beta} + \frac{1}{6 \log_a \beta} = \frac{\frac{1}{2 \log \beta}}{\log \alpha} + \frac{\frac{1}{3 \log \beta}}{\log \alpha} + \frac{\frac{1}{6 \log \beta}}{\log \alpha} = \\ &= \frac{\log_\beta \alpha}{2} + \frac{\log_\beta \alpha}{3} + \frac{\log_\beta \alpha}{6} = \log_\beta \alpha \end{aligned}$$

28.20 4)

$$\frac{\log_{a\beta} \alpha}{1 + \log_\beta \alpha} = \frac{\frac{\log_{a\beta} \alpha}{\log_{a\beta} \beta}}{\frac{1 + \log_{a\beta} \alpha}{\log_{a\beta} \beta}} = \frac{\log_{a\beta} \alpha}{\log_{a\beta} \beta + \log_{a\beta} \alpha} = \frac{\log_{a\beta} \alpha}{\log_{a\beta} a\beta} = \log_{a\beta} a$$

28.20 5)

$$\log_a \beta^2 \cdot \log_{a^2} \beta = 2 \log_a \beta \cdot \log_{a^2} \beta = 2 \log_a \beta \cdot \frac{\log_a \beta}{\log_a a^2} = \frac{2 (\log_a \beta)^2}{2} = (\log_a \beta)^2$$

28.20 6)

$$\log_a \theta \cdot \log_\beta \theta + \log_\beta \theta \cdot \log_\gamma \theta + \log_\gamma \theta \cdot \log_a \theta = \frac{\log \theta}{\log \alpha} \cdot \frac{\log \theta}{\log \beta} + \frac{\log \theta}{\log \beta} \cdot \frac{\log \theta}{\log \gamma} + \frac{\log \theta}{\log \gamma} \cdot \frac{\log \theta}{\log \alpha} =$$

$$= (\log \theta)^2 \left(\frac{1}{\log \alpha \log \beta} + \frac{1}{\log \beta \log \gamma} + \right) \frac{1}{\log \alpha \log \gamma} = (\log \theta)^2 \left(\frac{\log \alpha + \log \beta + \log \gamma}{\log \alpha \cdot \log \beta \cdot \log \gamma} \right) =$$

$$= \frac{(\log \theta) \cdot (\log \theta) \cdot \log(a\beta\gamma)}{\log \alpha \cdot \log \beta \cdot \log \gamma} = \frac{\log \theta}{\log \alpha} \cdot \frac{\log \theta}{\log \beta} \cdot \frac{\log(a\beta\gamma)}{\log \gamma} =$$

$$\begin{aligned}
& \text{τύπος} \\
& \text{αλλαγής} \\
& \beta \alpha \gamma \zeta \\
& = \log_a \theta \cdot \log_b \theta \cdot \frac{\log(\alpha \beta \gamma)}{\log \theta} \cdot \frac{\log \theta}{\log \gamma} = \log_a \theta \cdot \log_b \theta \cdot \frac{1}{\log_{ab\gamma} \theta} \cdot \log_\gamma \theta = \\
& = \frac{\log_a \theta \cdot \log_b \theta \cdot \log_\gamma \theta}{\log_{ab\gamma} \theta}
\end{aligned}$$

28.20 7)

$$\text{i) } \log_a \beta = \frac{\log_b \beta}{\log_b a} = \frac{1}{\log_b a}$$

$$\text{ii) } \alpha^{\frac{2}{\log_b a}} - 2\alpha^{\frac{1}{\log_b a}} \cdot \beta^{\frac{1}{\log_a \beta}} + \beta^{\frac{2}{\log_a \beta}}$$

$$= \alpha^{2\log_a \beta} - 2\alpha^{\log_a \beta} \cdot \beta^{\log_b \alpha} + \beta^{2\log_b \alpha} =$$

$$= \alpha^{\log_a \beta^2} - 2 \cdot \beta \cdot \alpha + \beta^{\log_b \alpha^2} =$$

$$= \beta^2 - 2\alpha\beta + \alpha^2 = (\alpha - \beta)^2$$

28.20 8)

$$\frac{\log_a \theta - \log_{\sqrt{ab}} \theta}{\log_{\sqrt{ab}} \theta - \log_b \theta}$$

$$\text{Είναι: } \log_{\sqrt{ab}} \theta = \frac{\log_{ab} \theta}{\log_{ab} \sqrt{ab}} = 2 \log_{ab} \theta$$

$$\text{Αριθμητικά: } \frac{\log_a \theta - \log_{\sqrt{ab}} \theta}{\log_{\sqrt{ab}} \theta - \log_b \theta} = \frac{\log_a \theta - 2 \log_{ab} \theta}{2 \log_{ab} \theta - \log_b \theta} =$$

$$\begin{aligned}
& \stackrel{(1)}{=} \frac{\log_a \theta - \frac{2 \log_a \theta \log_b \theta}{\log_a \theta + \log_b \theta}}{\frac{2 \log_a \theta \log_b \theta}{\log_a \theta + \log_b \theta} - \log_b \theta} = \frac{(\log_a \theta)^2 - \log_a \theta \log_b \theta}{\log_a \theta \log_b \theta - (\log_b \theta)^2} = \frac{\log_a \theta (\log_a \theta - \log_b \theta)}{\log_b \theta (\log_a \theta - \log_b \theta)} = \frac{\log_a \theta}{\log_b \theta}
\end{aligned}$$