28.20 1)

$$\beta'\ \text{μέλος}=\frac{\log_\alpha\theta\cdot\log_\beta\theta}{\log_\alpha\theta+\log_\beta\theta}=\frac{\dfrac{\log_{\alpha\beta}\theta}{\log_{\alpha\beta}\alpha}\cdot\dfrac{\log_{\alpha\beta}\theta}{\log_{\alpha\beta}\beta}}{\dfrac{\log_{\alpha\beta}\theta}{\log_{\alpha\beta}\alpha}+\dfrac{\log_{\alpha\beta}\theta}{\log_{\alpha\beta}\beta}}=\frac{\log^2_{\alpha\beta}\theta}{\log_{\alpha\beta}\alpha\cdot\log_{\alpha\beta}\beta}=$$

$$=\frac{\log^{\cancel{2}}_{\alpha\beta}\theta}{\cancel{\log_{\alpha\beta}\alpha\cdot\log_{\alpha\beta}\beta}}=\frac{\log_{\alpha\beta}\theta}{\cancel{\log_{\alpha\beta}\theta}\dfrac{\log_{\alpha\beta}\beta+\log_{\alpha\beta}\alpha}{\cancel{\log_{\alpha\beta}\alpha\cdot\log_{\alpha\beta}\beta}}}=\frac{\log_{\alpha\beta}\theta}{\log_{\alpha\beta}\beta+\log_{\alpha\beta}\alpha}=\frac{\log_{\alpha\beta}\theta}{\log_{\alpha\beta}\alpha\beta}=\frac{\log_{\alpha\beta}\theta}{1}=\log_{\alpha\beta}\theta=\alpha'\ \text{μέλος}$$

28.20 2)

$$\log_{\alpha^6}\beta^4\cdot\log_{\beta^2}\alpha^3=\frac{\log_{\beta^2}\beta^4}{\log_{\beta^2}\alpha^6}\cdot\log_{\beta^2}\alpha^3=\frac{2\cdot\cancel{\log}^{1}_{\beta^2}\beta^2}{2\log_{\beta^2}\alpha^3}\cdot\log_{\beta^2}\alpha^3=1$$

28.20 3)

$$\frac{1}{\log_\alpha\beta^2}+\frac{1}{\log_\alpha\beta^3}+\frac{1}{\log_\alpha\beta^6}=\frac{1}{2\log_\alpha\beta}+\frac{1}{3\log_\alpha\beta}+\frac{1}{6\log_\alpha\beta}=\frac{\dfrac{1}{2\log_\beta\beta}}{\dfrac{\log_\beta\alpha}{}}+\frac{\dfrac{1}{3\log_\beta\beta}}{\dfrac{\log_\beta\alpha}{}}+\frac{\dfrac{1}{6\log_\beta\beta}}{\dfrac{\log_\beta\alpha}{}}=$$

$$=\frac{\log_\beta\alpha}{2}+\frac{\log_\beta\alpha}{3}+\frac{\log_\beta\alpha}{6}=\log_\beta\alpha$$

28.20 4)

$$\frac{\log_\beta\alpha}{1+\log_\beta\alpha}=\frac{\dfrac{\log_{\alpha\beta}\alpha}{\log_{\alpha\beta}\beta}}{\dfrac{1+\log_{\alpha\beta}\alpha}{\log_{\alpha\beta}\beta}}=\frac{\log_{\alpha\beta}\alpha}{\log_{\alpha\beta}\beta+\log_{\alpha\beta}\alpha}=\frac{\log_{\alpha\beta}\alpha}{\log_{\alpha\beta}\alpha\beta}=\log_{\alpha\beta}\alpha$$

28.20 5)

$$\log_\alpha\beta^2\cdot\log_{\alpha^2}\beta=2\log_\alpha\beta\cdot\log_{\alpha^2}\beta=2\log_\alpha\beta\cdot\frac{\log_\alpha\beta}{\log_\alpha\alpha^2}=\frac{2(\log_\alpha\beta)^2}{2}=(\log_\alpha\beta)^2$$

28.20 6)

$$\log_\alpha\theta\cdot\log_\beta\theta+\log_\beta\theta\cdot\log_\gamma\theta+\log_\gamma\theta\cdot\log_\alpha\theta=\frac{\log\theta}{\log\alpha}\cdot\frac{\log\theta}{\log\beta}+\frac{\log\theta}{\log\beta}\cdot\frac{\log\theta}{\log\gamma}+\frac{\log\theta}{\log\gamma}\cdot\frac{\log\theta}{\log\alpha}=$$

$$=(\log\theta)^2\left(\frac{1}{\log\alpha\log\beta}+\frac{1}{\log\beta\log\gamma}+\right)\frac{1}{\log\alpha\log\gamma}=(\log\theta)^2\left(\frac{\log\alpha+\log\beta+\log\gamma}{\log\alpha\cdot\log\beta\cdot\log\gamma}\right)=$$

$$=\frac{(\log\theta)\cdot(\log\theta)\cdot\log(\alpha\beta\gamma)}{\log\alpha\cdot\log\beta\cdot\log\gamma}=\frac{\log\theta}{\log\alpha}\cdot\frac{\log\theta}{\log\beta}\cdot\frac{\log(\alpha\beta\gamma)}{\log\gamma}=$$

τύπος αλλαγής βάσης

$$= \log_\alpha\theta\cdot\log_\beta\theta\cdot\frac{\log(\alpha\beta\gamma)}{\log\theta}\cdot\frac{\log\theta}{\log\gamma} = \log_\alpha\theta\cdot\log_\beta\theta\cdot\frac{1}{\log_{\alpha\beta\gamma}\theta}\cdot\log_\gamma\theta =$$

$$= \frac{\log_\alpha\theta\cdot\log_\beta\theta\cdot\log_\gamma\theta}{\log_{\alpha\beta\gamma}\theta}$$

28.20 7)

i) $\log_\alpha\beta = \frac{\log_\beta\beta}{\log_\beta\alpha} = \frac{1}{\log_\beta\alpha}$

ii) $\alpha^{\frac{2}{\log_\beta\alpha}} - 2\alpha^{\frac{1}{\log_\beta\alpha}}\cdot\beta^{\frac{1}{\log_\alpha\beta}} + \beta^{\frac{2}{\log_\alpha\beta}}$

$$= \alpha^{2\log_\alpha\beta} - 2\alpha^{\log_\alpha\beta}\cdot\beta^{\log_\beta\alpha} + \beta^{2\log_\beta\alpha} =$$

$$= \alpha^{\log_\alpha\beta^2} - 2\cdot\beta\cdot\alpha + \beta^{\log_\beta\alpha^2} =$$

$$= \beta^2 - 2\alpha\beta + \alpha^2 = (\alpha-\beta)^2$$

28.20 8)

$$\frac{\log_\alpha\theta - \log_{\sqrt{\alpha\beta}}\theta}{\log_{\sqrt{\alpha\beta}}\theta - \log_\beta\theta}$$

Είναι : $\log_{\sqrt{\alpha\beta}}\theta = \frac{\log_{\alpha\beta}\theta}{\log_{\alpha\beta}\sqrt{\alpha\beta}} = 2\log_{\alpha\beta}\theta$

Άρα : $\frac{\log_\alpha\theta - \log_{\sqrt{\alpha\beta}}\theta}{\log_{\sqrt{\alpha\beta}}\theta - \log_\beta\theta} = \frac{\log_\alpha\theta - 2\log_{\alpha\beta}\theta}{2\log_{\alpha\beta}\theta - \log_\beta\theta} =$

$$\overset{(1)}{=} \frac{\log_\alpha\theta - \frac{2\log_\alpha\theta\log_\beta\theta}{\log_\alpha\theta+\log_\beta\theta}}{\frac{2\log_\alpha\theta\log_\beta\theta}{\log_\alpha\theta+\log_\beta\theta} - \log_\beta\theta} = \frac{(\log_\alpha\theta)^2 - \log_\alpha\theta\log_\beta\theta}{\log_\alpha\theta\log_\beta\theta - (\log_\beta\theta)^2} = \frac{\log_\alpha\theta(\log_\alpha\theta - \log_\beta\theta)}{\log_\beta\theta(\log_\alpha\theta - \log_\beta\theta)} = \frac{\log_\alpha\theta}{\log_\beta\theta}$$