28.19 1)

$$\log_8 12=\frac{\log_3 12}{\log_3 8}=\frac{\log_3(4\cdot 3)}{\log_3 2^3}=\frac{\log_3 4+\log_3 3}{3\log_3 2}=\frac{\log_3 2^2+\log_3 3}{3\log_3 2}=\frac{2\log_3 2+\log_3 3}{3\log_3 2}\overset{\substack{\log_3 2=\alpha\\ \log_3 3=1}}{=}\frac{2\alpha+1}{3\alpha}$$

28.19 2)

$$\log_9 24=\frac{\log_2 24}{\log_2 9}=\frac{\log_2(8\cdot 3)}{\log_2 3^2}=\frac{\log_2 8+\log_2 3}{2\log_2 3}$$

28.19 3)

$$\log_{15} 45=\frac{\log_3 45}{\log_3 15}=\frac{\log_3(9\cdot 5)}{\log_3(3\cdot 5)}=\frac{\log_3 9+\log_3 5}{\log_3 3+\log_3 5}=\frac{\log_3 3^2+\log_3 5}{\log_3 3+\log_3 5}=\frac{2+\alpha}{1+\alpha}$$

28.19 4)

$$\log_{16} 28=\frac{\log_2 28}{\log_2 16}=\frac{\log_2(4\cdot 7)}{\log_2 2^4}=\frac{\log_2 4+\log_2 7}{4\log_2 2}=\frac{2+\alpha}{4}$$

28.19 5)

$$\log_2 75=\frac{\log 75}{\log 2}=\frac{\log\left(3\cdot 5^2\right)}{\log\frac{10}{5}}=\frac{\log 3+2\log 5}{\log 10-\log 5}=\frac{\log 3+2\log 5}{1-\log 5}=\frac{\beta+2\alpha}{1-\alpha}$$