

Β ΛΥΚΕΙΟΥ ΑΛΓΕΒΡΑ

28.15 1)

$$\alpha) 27^{1-\frac{1}{3}\log_3 2} = \frac{27^1}{27^{\frac{1}{3}\log_3 2}} = \frac{27}{(3^3)^{\frac{1}{3}\log_3 2}} = \frac{27}{3^{\cancel{3}^1 \log_3 2}} = \frac{27}{3^{\log_3 2}} = \frac{27}{3^{\log_3 2} = 2} = \frac{27}{2}$$

$$\beta) 4^{2+\frac{1}{2}\log_2 3} = 4^2 \cdot 4^{\frac{1}{2}\log_2 3} = 16 \cdot (2^2)^{\frac{1}{2}\log_2 3} = 16 \cdot (2^2)^{\frac{1}{2}\log_2 3} = 16 \cdot 2^{\cancel{2}^1 \log_2 3} = 16 \cdot 2^{\log_2 3} = 16 \cdot 2^{\log_2 3} = 16 \cdot 3 = 48$$

$$\gamma) 100^{\frac{1}{3}\log_8 \frac{1}{2}} = \frac{100^{\frac{1}{3}\log_8}}{100^{\frac{1}{2}}} = \frac{100^{\log_8 \frac{1}{3}}}{\sqrt{100}} = \frac{(10^2)^{\log_8 \frac{1}{3}}}{10} = \frac{10^{2\log_8}}{10} = \frac{10^{\log_8 2^2}}{10} = \frac{10^{\log_8 4}}{10} = \frac{10^{\log_8 4}}{10} = \frac{10^{\log_8 4}}{10} = \frac{4}{10} = \frac{2}{5}$$

$$\delta) \left(\frac{1}{\sqrt{e}}\right)^{2\ln 3 - \ln 64} = \left(e^{-\frac{1}{2}}\right)^{\ln 3^2 - \ln 64} = \left(e^{-\frac{1}{2}}\right)^{\ln 9 - \ln 64} = \left(e^{-\frac{1}{2}}\right)^{\ln \frac{9}{64}} = e^{-\frac{1}{2} \ln \frac{9}{64}} = e^{\ln \left(\frac{9}{64}\right)^{-\frac{1}{2}}} = \left(\frac{9}{64}\right)^{-\frac{1}{2}} = \left(\frac{64}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

28.15 2)

$$16^{\log_2 3} = (2^4)^{\log_2 3} = 2^{\log_2 3^4} = 3^4 = 81$$

28.15 3)

$$1000^{\log 2} = 10^{3\log 2} = 10^{\log 2^3} = 10^{\log 8} = 8$$

28.15 4)

$$3^{\log_{\sqrt{3}} 7} = \left(\sqrt{3}^2\right)^{\log_{\sqrt{3}} 7} = \sqrt{3}^{\log_{\sqrt{3}} 7^2} = 7^2 = 49$$

28.15 5)

$$\sqrt{7}^{\log_7 25} = \left(7^{\frac{1}{2}}\right)^{\log_7 25} = 7^{\log_7 25^{\frac{1}{2}}} = 25^{\frac{1}{2}} = 5$$

28.15 6)

$$\sqrt{10}^{\log 9} = 10^{\frac{1}{2}\log 9} = 10^{\log \sqrt{9}} = \sqrt{9} = 3$$

28.15 7)

$$\sqrt{e}^{\ln 16} = e^{\frac{1}{2}\ln 16} = e^{\ln 4} = 4$$

28.15 8)

$$\left(\frac{1}{e}\right)^{\ln 3} = e^{-\ln 3} = e^{\ln 3^{-1}} = \frac{1}{3}$$

28.15 9)

$$36^{1-\log_6 2} = \frac{36}{36^{\log_6 2}} = \frac{36}{(6^2)^{\log_6 2}} = \frac{36}{6 \log_6 2^2} = \frac{36}{2^2} = 9$$

28.15 10)

$$25^{1+\log_5 3} = 25 \cdot 25^{\log_5 3} = 25 \cdot (5^2)^{\log_5 3} = 25 \cdot 5^{\log_5 3^2} = 25 \cdot 3^2 = 225$$

28.15 11)

$$100^{\left(\frac{1}{2}-\log \sqrt[4]{4}\right)} = 10^{2\left(\frac{1}{2}-\log \sqrt[4]{4}\right)} = 10^{1-2\log \sqrt[4]{4}} = \frac{10}{10^{\log \sqrt[4]{4}}} = \frac{10}{\sqrt[4]{4}} = \frac{10}{2} = 5$$

28.15 12)

$$100^{\frac{1}{2} \log 9 - \log 2} = 10^{2 \left(\frac{1}{2} \log 9 - \log 2 \right)} = 10^{\log 9 - \log 4} = 10^{\log \frac{9}{4}} = \frac{9}{4}$$

28.15 13)

$$49^{1 + \log_7 2} = 49 \cdot 49^{\log_7 2} = 49 \cdot (7^2)^{\log_7 2} = 49 \cdot 7^{\log_7 2^2} = 49 \cdot 2^2 = 196$$

28.15 14)

$$\sqrt{10^{2 + \frac{1}{2} \log 16}} = \sqrt{10^2 \cdot 10^{\log \sqrt{16}}} = \sqrt{100 \cdot 10^{\log 4}} = \sqrt{4 \cdot 100} = 20$$

28.15 15)

$$\frac{e^{2+3 \ln 2}}{100^{\log e}} = \frac{e^2 \cdot e^{\ln 2^3}}{10^{2 \log e}} = \frac{e^2 \cdot e^{\ln 8}}{10^{\log e^2}} = \frac{e^2 8}{e^2} = 8$$

28.15 16)

$$\frac{5^{1 + \log_5 18}}{4^{\log_2 3}} = \frac{5 \cdot 5^{\log_5 18}}{(2^2)^{\log_2 3}} = \frac{5 \cdot 18}{2^{\log_2 3^2}} = \frac{5 \cdot 18}{9} = 10$$

28.15 17)

$$\begin{aligned} 49^{1 - \log_7 2} + 5^{-\log_5 4} &= \frac{49}{49^{\log_7 2}} + 5^{\log_5 4^{-1}} = \\ &= \frac{49}{7^{2 \log_7 2}} + \frac{1}{4} = \frac{49}{4} + \frac{1}{4} = \frac{50}{4} = \frac{25}{2} \end{aligned}$$

28.15 18)

$$3^{3 - \log_3 5} \cdot e^{\ln 5} = 3^3 \cdot 3^{-\log_3 5} \cdot 5 = 27 \cdot 3^{\log_3 5^{-1}} \cdot 5 = 27 \cdot 5^{-1} \cdot 5 = \frac{1}{5} \cdot 5 = 1$$

28.15 19)

$$\begin{aligned} 2^{\log_4 (2 - \sqrt{3})^4} + 3^{\log_3 (2 + \sqrt{3})^2} &= 4^{\frac{1}{2} \log_4 (2 - \sqrt{3})^4} + (2\sqrt{3})^2 = \\ &= 4^{\log_4 (2 - \sqrt{3})^2} + (2 + \sqrt{3})^2 = (2 - \sqrt{3})^2 + (2 + \sqrt{3})^2 = \\ &= 4 - 4\sqrt{3} + 3 + 4 + 4\sqrt{3} + 3 = 14 \end{aligned}$$

28.15 20)

$$(e^2)^{\ln \sqrt{e}} + \sqrt{e^{\frac{1}{2} \ln e^2 + 1}} = e^{\ln \sqrt{e^2}} + \sqrt{e^{\ln e} \cdot e} = e^{\ln e} + \sqrt{e \cdot e} = e + \sqrt{e^2} = 2e$$

28.15 21)

$$\left(\frac{1}{e}\right)^{2 \ln 5 - \ln 50} = \frac{1}{e^{\ln 25 - \ln 50}} = \frac{1}{e^{\ln \frac{25}{50}}} = \frac{1}{e^{\ln \frac{1}{2}}} = \frac{1}{\frac{1}{2}} = 2$$

28.15 22)

$$\left(\frac{1}{64}\right)^{\log_8 3 - \log_8 12} = \left(\frac{1}{64}\right)^{\log_8 \frac{3}{12}} = \frac{1}{(8^2)^{\log_8 \frac{3}{12}}} = \frac{1}{8^{\log_8 \left(\frac{3}{12}\right)^2}} = \frac{1}{\left(\frac{3}{12}\right)^2} = \frac{1}{\left(\frac{1}{4}\right)^2} = 16$$

28.15 **23)**

$$\frac{\sqrt{e}^{\ln 8} \cdot e^{\log 1000}}{e^{3+\ln \sqrt{2}}} = \frac{e^{\frac{1}{2}\ln 8} \cdot e^{\log 10^3}}{e^3 \cdot e^{\ln \sqrt{2}}} = \frac{e^{\ln 8 \frac{1}{2}} \cdot e^3}{e^3 \cdot e^{\ln \sqrt{2}}} = \frac{\sqrt{8}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}} = 2$$

28.15 **24)**

$$\left(\frac{\sqrt{e}}{e^3}\right)^{\ln 4} = \left(\frac{e^{\frac{1}{2}}}{e^3}\right)^{\ln 4} = \left(e^{-\frac{5}{2}}\right)^{\ln 4} = e^{\ln 4 \cdot \frac{1}{2}} = 4^{-\frac{5}{2}} = \frac{1}{\sqrt{4^5}} = \frac{1}{\sqrt{2^{10}}} = \frac{1}{2^5} = \frac{1}{32}$$