28.12 1)

$$\log_6\left(12+6\sqrt{3}\right)+2\log_6\left(3-\sqrt{3}\right)=\log_6\left(12+6\sqrt{3}\right)+\log_6\left(3-\sqrt{3}\right)^2=$$

$$=\log_6\left(12+6\sqrt{3}\right)+\log_6\left(9-6\sqrt{3}+3\right)=\log_6\left(12+6\sqrt{3}\right)+\log_6\left(12-6\sqrt{3}\right)=$$

$$=\log_6\left[\left(12+6\sqrt{3}\right)\left(12-6\sqrt{3}\right)\right]=\log_6\left(12^2-6^2\cdot\sqrt{3}^2\right)=$$

$$=\log_6\left(144-108\right)=\log_6 36=\log_6 6^2=2$$

28.12 2)

$$\log_7\left(11-6\sqrt{2}\right)+2\log_7\left(3+\sqrt{2}\right)=\log_7\left(11-6\sqrt{2}\right)+\log_7\left(3+\sqrt{2}\right)^2=$$

$$=\log_7\left(11-6\sqrt{2}\right)+\log_7\left(9+6\sqrt{2}+2\right)=\log_7\left(11-6\sqrt{2}\right)+\log_7\left(11+6\sqrt{2}\right)=$$

$$=\log_7\left[\left(11-6\sqrt{2}\right)\left(11+6\sqrt{2}\right)\right]=\log_7\left(11^2-6^2\cdot\sqrt{2}^2\right)=$$

$$=\log_7\left(121-72\right)=\log_7 49=\log_7 7^2=2$$

28.12 3)

$$2\log_5\left(2-\sqrt{3}\right)+\log_5\left(7+4\sqrt{3}\right)=\log_5\left(2-\sqrt{3}\right)^2+\log_5\left(7+4\sqrt{3}\right)=$$

$$=\log_5\left(4-4\sqrt{3}+3\right)+\log_5\left(7+4\sqrt{3}\right)=\log_5\left(7-4\sqrt{3}\right)+\log_5\left(7+4\sqrt{3}\right)=$$

$$=\log_5\left[\left(7-4\sqrt{3}\right)\left(7+4\sqrt{3}\right)\right]=\log_5\left(7^2-4^2\cdot\sqrt{3}^2\right)=\log_5 1=0$$

28.12 4)

$$\log 2+\log\left(\sqrt{3}+1\right)=\log\left(1+\sqrt{2-\sqrt{3}}\right)+\log\left(1-\sqrt{2-\sqrt{3}}\right)=$$

$$=\log\left(2\left(\sqrt{3}+1\right)\right)+\log\left[\left(1+\sqrt{2-\sqrt{3}}\right)\left(1-\sqrt{2-\sqrt{3}}\right)\right]=$$

$$=\log\left(2\left(\sqrt{3}+1\right)\right)+\log\left(1-\left(2-\sqrt{3}\right)\right)=\log\left(2\left(\sqrt{3}+1\right)\right)+\log\left(-1+\sqrt{3}\right)=$$

$$=\log\left(2\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)\right)=\log\left(2\left(3-1\right)\right)=\log 2^2=2\log 2$$

28.12 5)

$$\log 2+\log\left(2+\sqrt{2}\right)+\log\left(2+\sqrt{2+\sqrt{2}}\right)+\log\left(2-\sqrt{2+\sqrt{2}}\right)=$$

$$=2\log 2=\log\left(2\left(2+\sqrt{3}\right)\right)+\log\left[\left(2+\sqrt{2+\sqrt{2}}\right)\left(2-\sqrt{2+\sqrt{2}}\right)\right]=$$

$$=\log\left(2\left(2+\sqrt{2}\right)\right)+\log\left(4-\left(2+\sqrt{2}\right)\right)=\log\left(2\left(2+\sqrt{3}\right)\right)+\log\left(2-\sqrt{2}\right)=$$

$$= \log\left[2\left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right)\right] = \log\left[2\left(2^2-2\right)\right] = \log 2^2 = 2\log 2$$

28.12 6)

$$\frac{1}{2}\log 33 + \frac{1}{2}\log\left(6+\sqrt{3}\right) + \frac{1}{2}\log\left(3+\sqrt{3+\sqrt{3}}\right) + \frac{1}{2}\log\left(3-\sqrt{3+\sqrt{3}}\right) =$$

$$= \log\sqrt{33} + \log\left(\sqrt{6+\sqrt{3}}\right) + \frac{1}{2}\log\left[\left(3+\sqrt{3+\sqrt{3}}\right)\left(3-\sqrt{3+\sqrt{3}}\right)\right] =$$

$$= \log\sqrt{33} + \log\left(\sqrt{6+\sqrt{3}}\right) + \frac{1}{2}\log\left(9-\left(3+\sqrt{3}\right)\right) =$$

$$= \log\sqrt{33} + \frac{1}{2}\log\left(6+\sqrt{3}\right) + \frac{1}{2}\log\left(6-\sqrt{3}\right) =$$

$$= \log\sqrt{33} + \frac{1}{2}\log\left[\left(6+\sqrt{3}\right)\left(6-\sqrt{3}\right)\right] = \log\sqrt{33} + \frac{1}{2}\log\left(6^2-3\right) =$$

$$= \log\sqrt{33} + \frac{1}{2}\log 33 = \log\sqrt{33} + \log\sqrt{33} = \log 33$$