

Β ΛΥΚΕΙΟΥ ΑΛΓΕΒΡΑ

27.8 1)

$$\alpha) 7^{x^2-2x+1} = 5^{(x-1)^2} \Rightarrow 7^{(x-1)^2} = 5^{(x-1)^2} \Rightarrow \left(\frac{7}{5}\right)^{(x-1)^2} = \left(\frac{7}{5}\right)^0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1$$

$$\beta) 2^{x+1} \cdot 5^x = 200 \Rightarrow 2 \cdot 2^x \cdot 5^x = 200 \Rightarrow (2 \cdot 5)^x = 100 \Rightarrow 10^x = 10^2 \Rightarrow x = 2$$

$$\gamma) 6^{2x+4} = 27^x \cdot 2^{x+8} \Rightarrow (2 \cdot 3)^{2x+4} = 3^{3x} \cdot 2^{x+8} \Rightarrow 2^{2x+4} \cdot 3^{2x+4} = 3^{3x} \cdot 2^{x+8} \Rightarrow$$

$$\Rightarrow 2^{2x+4-x-8} = 3^{3x-2x-4} \Rightarrow 2^{x-4} = 3^{x-4} \Rightarrow \left(\frac{2}{3}\right)^{x-4} = 1 \Rightarrow \left(\frac{2}{3}\right)^{x-4} = \left(\frac{2}{3}\right)^0 \Rightarrow x-4 = 0 \Rightarrow x = 4$$

$$\delta) 25^x - 7^x - 35 \cdot 5^{2x} + 5 \cdot 7^{x+1} = 0 \Rightarrow 25^x - 7^x - 35 \cdot 25^x + 5 \cdot 7 \cdot 7^x = 0 \Rightarrow$$

$$\Rightarrow -34 \cdot 25^x + 34 \cdot 7^x = 0 \Rightarrow 25^x = 7^x \Rightarrow \left(\frac{25}{7}\right)^x = 1 \Rightarrow \left(\frac{25}{7}\right)^x = \left(\frac{25}{7}\right)^0 \Rightarrow x = 0$$

$$\epsilon) 3 \cdot 4^x + \frac{1}{3} \cdot 9^{x+2} = 6 \cdot 4^{x+1} - \frac{1}{2} \cdot 9^{x+1} \Rightarrow 2 \cdot 3^{2x} + \frac{9}{3} \cdot 9^{x+1} = 6 \cdot 4 \cdot 4^x - \frac{1}{2} \cdot 9^{x+1} \Rightarrow$$

$$\Rightarrow 3 \cdot 2^{2x} + 3 \cdot 9^{x+1} = 24 \cdot 2^{2x} - \frac{1}{2} \cdot 9^{x+1} \Rightarrow 3 \cdot 2^{2x} + 3 \cdot 9 \cdot 9^x = 24 \cdot 2^{2x} - \frac{9}{2} \cdot 9^x \Rightarrow$$

$$\Rightarrow 3 \cdot 2^{2x} + 27 \cdot 9^x = 24 \cdot 2^{2x} - \frac{9}{2} \cdot 3^{2x} \Rightarrow 21 \cdot 2^{2x} = \frac{63}{2} \cdot 3^{2x} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{3}\right)^{2x} = \frac{3}{2} \Rightarrow \left(\frac{2}{3}\right)^{2x} = \left(\frac{2}{3}\right)^{-1} \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$$

27.8 2)

$$3^{x^2-4x+4} = 4^{(x-2)^2} \Rightarrow 3^{(x-2)^2} = 4^{(x-2)^2} \Rightarrow \left(\frac{3}{4}\right)^{(x-2)^2} = 1 \Rightarrow$$

$$\Rightarrow \left(\frac{3}{4}\right)^{(x-2)^2} = \left(\frac{3}{4}\right)^0 \Rightarrow (x-2)^2 = 0 \Rightarrow x = 2$$

27.8 3)

$$3^{|x|-2} = 5^{|x|-2} \Rightarrow \left(\frac{3}{5}\right)^{|x|-2} = 1 \Rightarrow \left(\frac{3}{5}\right)^{|x|-2} = \left(\frac{3}{5}\right)^0 \Rightarrow |x|-2 = 0 \Rightarrow |x| = 2 \Rightarrow x = \pm 2$$

27.8 4)

$$2^{x+1} \cdot 5^x = 200 \Rightarrow 2 \cdot 2^x \cdot 5^x = 200 \Rightarrow (2 \cdot 5)^x = 100 \Rightarrow 10^x = 10^2 \Rightarrow x = 2$$

27.8 5)

$$3^{2x-1} \cdot 5^{3x+2} = \frac{9}{5} \cdot 5^{2x} \cdot 3^{3x} \Rightarrow 3^{2x-1} \cdot 5^{3x+2} = 5^{2x-1} \cdot 3^{3x+2} \Rightarrow$$

$$\Rightarrow \left(\frac{3}{5}\right)^{2x-1} = \left(\frac{3}{5}\right)^{3x+2} \Rightarrow 2x-1 = 3x+2 \Rightarrow x = -3$$

27.8 6)

$$\sqrt{3^x} \cdot \sqrt{5^x} = 225 \Rightarrow 3^{\frac{x}{2}} \cdot 5^{\frac{x}{2}} = 225 \Rightarrow$$

$$\Rightarrow (3 \cdot 5)^{\frac{x}{2}} = 15^2 \Rightarrow 15^{\frac{x}{2}} = 15^2 \Rightarrow \frac{x}{2} = 2 \Rightarrow x = 4$$

27.8 7)

$$5 \cdot 2^x + 4 \cdot 3^x = 8 \cdot 3^x - 4 \cdot 2^x \Rightarrow \\ \Rightarrow 5 \cdot 2^x + 4 \cdot 3^x - 8 \cdot 3^x + 4 \cdot 2^x = 0 \Rightarrow$$

$$\Rightarrow 9 \cdot 2^x - 4 \cdot 3^x = 0 \Rightarrow \frac{2^x}{3^x} = \frac{4}{9} \Rightarrow \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^2 \Rightarrow x = 2$$

27.8 8)

$$4 \cdot 5^x - 2^{x+1} = 5^{x+1} - 3 \cdot 2^x \Rightarrow 2^2 \cdot 5^x - 2^x \cdot 2 = 5^x \cdot 5 - 3 \cdot 2^x \Rightarrow$$

$$\Rightarrow 5^x - 2^x = 0 \Rightarrow 5^x = 2^x \Rightarrow \left(\frac{5}{2}\right)^x = 1 \Rightarrow \left(\frac{5}{2}\right)^x = \left(\frac{5}{2}\right)^0 \Rightarrow x = 0$$

27.8 9)

$$5^{2x} - 3^x - 15 \cdot 25^x + 15 \cdot 3^x = 0 \Rightarrow 5^{2x} - 3^x - 15 \cdot 5^{2x} + 15 \cdot 3^x = 0 \Rightarrow$$

$$\Rightarrow 14 \cdot 5^{2x} = 14 \cdot 3^x \Rightarrow 5^{2x} = 3^x \Rightarrow 25^x = 3^x \Rightarrow$$

$$\Rightarrow \left(\frac{25}{3}\right)^x = 1 \Rightarrow \left(\frac{25}{3}\right)^x = \left(\frac{25}{3}\right)^0 \Rightarrow x = 0$$

27.8 10)

$$8 \cdot 2^{x-2} + \frac{2^x}{4} = 6^{x-1} - 3 \cdot 6^{x-2} \Rightarrow 8 \cdot 2^{x-2} + 2^{x-2} = 6 \cdot 6^{x-2} - 3 \cdot 6^{x-2} \Rightarrow$$

$$\Rightarrow 9 \cdot 2^{x-2} = 3 \cdot 6^{x-2} \Rightarrow 3 \cdot 2^{x-2} = 6^{x-2} \Rightarrow$$

$$\Rightarrow \left(\frac{6}{2}\right)^{x-2} = 3 \Rightarrow 3^{x-2} = 3 \Rightarrow x-2=1 \Rightarrow x=3$$

27.8 11)

$$7 \cdot 3^{x+1} - 5^{x+2} = 3^{x+4} - 5^{x+3} \Rightarrow 7 \cdot 3^{x+1} - 5 \cdot 5^{x+1} = 27 \cdot 3^{x+1} - 25 \cdot 5^{x+1} \Rightarrow$$

$$\Rightarrow 7 \cdot 3^{x+1} - 5 \cdot 5^{x+1} = 27 \cdot 3^{x+1} - 25 \cdot 5^{x+1} \Rightarrow 20 \cdot 3^{x+1} = 20 \cdot 5^{x+1} \Rightarrow$$

$$\Rightarrow \left(\frac{3}{5}\right)^{x+1} = 1 \Rightarrow \left(\frac{3}{5}\right)^{x+1} = \left(\frac{3}{5}\right)^0 \Rightarrow x+1=0 \Rightarrow x=-1$$

27.8 12)

$$2 \cdot 9^{x+1} - 3 \cdot 2^{2x} = 5 \cdot 4^x - 9 \cdot 3^{2x} \Rightarrow 2 \cdot 9 \cdot 9^x - 3 \cdot 2^{2x} = 5 \cdot 2^{2x} - 9 \cdot 3^{2x} \Rightarrow$$

$$\Rightarrow 18 \cdot 3^{2x} - 3 \cdot 2^{2x} = 5 \cdot 2^{2x} - 9 \cdot 3^{2x} \Rightarrow 27 \cdot 3^{2x} = 8 \cdot 2^{2x} \Rightarrow$$

$$\Rightarrow 3^3 \cdot 3^{2x} = 2^3 \cdot 2^{2x} \Rightarrow 3^{2x+3} = 2^{2x+3} \Rightarrow$$

$$\Rightarrow \left(\frac{3}{2}\right)^{2x+3} = 1 \Rightarrow \left(\frac{3}{2}\right)^{2x+3} = \left(\frac{3}{2}\right)^0 \Rightarrow 2x+3=0 \Rightarrow x=-\frac{3}{2}$$