

Β ΛΥΚΕΙΟΥ ΑΛΓΕΒΡΑ

27.7 1)

α) $2 \cdot 2^{2x} - 3 \cdot 2^x + 1 = 0 \Rightarrow 2 \cdot (2^x)^2 - 3 \cdot 2^x + 1 = 0$ οπότε θέτω $2^x = \omega$ και η εξίσωση γράφεται :

$$2\omega^2 - 3\omega + 1 = 0 \quad \Delta = 1 \quad \omega_{1,2} = \frac{3 \pm 1}{4} \begin{matrix} \nearrow \omega_1 = \frac{4}{4} = 1 \\ \searrow \omega_2 = \frac{2}{4} = \frac{1}{2} \end{matrix}$$

$$\text{Άρα: } \omega = 1 \Rightarrow 2^x = 1 = 2^0 \Rightarrow x = 0 \quad \text{ή} \quad \omega = \frac{1}{2} \Rightarrow 2^x = \frac{1}{2} \Rightarrow 2^x = 2^{-1} \Rightarrow x = -1$$

β) $3^{2x+5} - 3^{x+2} = 2 \Rightarrow 3^{2x} \cdot 3^5 - 3^x \cdot 3^2 = 2 \Rightarrow 243 \cdot 3^{2x} - 9 \cdot 3^x - 2 = 0$

θέτω $3^x = \omega$ και η εξίσωση γράφεται : $243\omega^2 - 9\omega - 2 = 0$

$$\Delta = 2025 \quad \omega_{1,2} = \frac{9 \pm 45}{486} \begin{matrix} \nearrow \omega_1 = \frac{9+45}{486} = \frac{54}{486} = \frac{1}{9} \\ \searrow \omega_2 = \frac{9-45}{486} = -\frac{36}{486} = -\frac{2}{27} \end{matrix}$$

$$\text{Άρα: } \omega = \frac{1}{9} \Rightarrow 3^x = 3^{-2} \Rightarrow x = -2 \quad \text{ή} \quad \omega = -\frac{2}{27} \Rightarrow 3^x = -\frac{2}{27} \text{ αδύνατη (διότι } 3^x > 0)$$

27.7 2)

$9^{2x} - 2 \cdot 9^x - 3 = 0$ θέτω $9^x = \omega$ και η εξίσωση γράφεται :

$$\omega^2 - 2\omega - 3 = 0$$

$$\Delta = 16 \quad \omega_{1,2} = \frac{2 \pm 4}{2} \begin{matrix} \nearrow \omega_1 = 3 \\ \searrow \omega_2 = -\frac{1}{2} \end{matrix}$$

$$\text{Άρα: } * \omega = 3 \Rightarrow 9^x = 3 = 9^{\frac{1}{2}} \Rightarrow x = \frac{1}{2} \quad \text{ή}$$

$$* \omega = -\frac{1}{2} \Rightarrow 9^x = -\frac{1}{2} \text{ αδύνατη αφού } 9^x > 0 \text{ για κάθε } x \in \mathbb{R}$$

27.7 3)

$2^x - \frac{6}{2^x} = -1$ θέτω $2^x = \omega \neq 0$ και η εξίσωση γράφεται :

$$\omega - \frac{6}{\omega} = -1 \xrightarrow{\omega \neq 0} \omega^2 - 6 = -\omega \Rightarrow \omega^2 + \omega - 6 = 0$$

$$\Delta = 25$$

$$\omega_{1,2} = \frac{-1 \pm 5}{2} \begin{matrix} \nearrow x_1 = 2 \\ \searrow x_2 = -3 \end{matrix}$$

Άρα: * $\omega = 2 \Rightarrow 2^x = 2 \Rightarrow x = 1$ ή

* $\omega = -3 \Rightarrow 2^x = -3$ αδύνατη αφού $2^x > 0$ για κάθε $x \in \mathbb{R}$

27.7 4)

$5^x - 24 = \frac{25}{5^x}$ θέτω $5^x = \omega \neq 0$ και η εξίσωση γράφεται :

$$\omega - 24 = \frac{25}{\omega} \Rightarrow \omega^2 - 24\omega - 25 = 0$$

$$\Delta = 676 \quad \omega_{1,2} = \frac{24 \pm 26}{2} \begin{matrix} \nearrow \omega_1 = 25 \\ \searrow \omega_2 = -1 \end{matrix}$$

Άρα: * $\omega = 25 \Rightarrow 5^x = 5^2 \Rightarrow x = 2$ ή

* $\omega = -1 \Rightarrow 5^x = -1$ αδύνατη αφού $5^x > 0$ για κάθε $x \in \mathbb{R}$

27.7 5)

$3^x + \frac{1}{3^x} = \frac{10}{3}$ θέτω $3^x = \omega \neq 0$ και η εξίσωση γράφεται :

$$\omega + \frac{1}{\omega} = \frac{10}{3} \Rightarrow \omega^2 + 1 = \frac{10\omega}{3} \Rightarrow 3\omega^2 + 3 - 10\omega = 0 \Rightarrow 3\omega^2 - 10\omega + 3 = 0$$

$$\Delta = 64 \quad \omega_{1,2} = \frac{10 \pm 8}{6} \begin{matrix} \nearrow \omega_1 = \frac{1}{3} \\ \searrow \omega_2 = 3 \end{matrix}$$

Άρα: * $\omega = \frac{1}{3} \Rightarrow 3^x = \frac{1}{3} \Rightarrow 3^x = 3^{-1} \Rightarrow x = -1$ ή

* $\omega = 3 \Rightarrow 3^x = 3 \Rightarrow x = 1$

27.7 6)

$4^{\frac{2}{x}} - 5 \cdot 4^{\frac{1}{x}} + 4 = 0$ θέτω $4^{\frac{1}{x}} = \omega \neq 0$ και η εξίσωση γράφεται :

$$\omega^2 - 5\omega + 4 = 0$$

$$\Delta = 9 \quad \omega_{1,2} = \frac{5 \pm 3}{2} \begin{matrix} \nearrow \omega_1 = 4 \\ \searrow \omega_2 = 1 \end{matrix}$$

Άρα: * $\omega = 4 \Rightarrow 4^{\frac{1}{x}} = 4^1 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$ ή

* $\omega = 1 \Rightarrow 4^{\frac{1}{x}} = 4^0 \Rightarrow \frac{1}{x} = 0$ αδύνατη

27.7 7)

$2^{\frac{2}{x}} - 2^{\frac{1}{x}} = 2$ θέτω $2^{\frac{1}{x}} = \omega \neq 0$ και η εξίσωση γράφεται :

$$\omega^2 - \omega = 2 \Rightarrow \omega^2 - \omega - 2 = 0$$

$$\Delta = 9 \quad \omega_{1,2} = \frac{1 \pm 3}{2} \begin{matrix} \nearrow \omega_1 = 2 \\ \searrow \omega_2 = -1 \end{matrix}$$

$$\text{Άρα: } * \omega = 2 \Rightarrow 2^{\frac{1}{x}} = 2^1 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1 \quad \text{ή}$$

$$* \omega = -1 \Rightarrow 2^{\frac{1}{x}} = -1 \text{ αδύνατη αφού } 2^{\frac{1}{x}} > 0 \text{ για κάθε } x \in \mathbb{R}$$

27.7 8)

$$5^{\frac{2}{x}} - 6 \cdot 5^{\frac{1}{x}} + 5 = 0 \quad \text{θέτω } 5^{\frac{1}{x}} = \omega \text{ και η εξίσωση γράφεται :}$$

$$\omega^2 - 6\omega + 5 = 0$$

$$\Delta = 16 \quad \omega_{1,2} = \frac{6 \pm 4}{2} \begin{matrix} \nearrow \omega_1 = 5 \\ \searrow \omega_2 = 1 \end{matrix}$$

$$\text{Άρα: } * \omega = 1 \Rightarrow 5^{\frac{1}{x}} = 5^1 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1 \quad \text{ή}$$

$$* \omega = 5 \Rightarrow 5^{\frac{1}{x}} = 5^0 \Rightarrow \frac{1}{x} = 0 \text{ αδύνατη}$$

27.7 9)

$$3^{\frac{2}{x}} - 4 \cdot 3^{\frac{1}{x}} + 3 = 0 \quad \text{θέτω } 3^{\frac{1}{x}} = \omega \text{ και η εξίσωση γράφεται :}$$

$$\omega^2 - 4\omega + 3 = 0$$

$$\Delta = 4 \quad \omega_{1,2} = \frac{4 \pm 2}{2} \begin{matrix} \nearrow \omega_1 = 3 \\ \searrow \omega_2 = 1 \end{matrix}$$

$$\text{Άρα: } * \omega = 3 \Rightarrow 3^{\frac{1}{x}} = 3^1 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1 \quad \text{ή}$$

$$* \omega = 1 \Rightarrow 3^{\frac{1}{x}} = 3^0 \Rightarrow \frac{1}{x} = 0 \text{ αδύνατη}$$

27.7 10)

$$3\sqrt[3]{81} - 10\sqrt[3]{9} + 3 = 0, \quad x > 0$$

$$\Rightarrow 3 \cdot 81^{\frac{1}{3}} - 10 \cdot 9^{\frac{1}{3}} + 3 = 0 \Rightarrow 3 \cdot 9^{\frac{2}{3}} - 10 \cdot 9^{\frac{1}{3}} + 3 = 0$$

$$\text{θέτω } 9^{\frac{1}{3}} = \omega \text{ και η εξίσωση γράφεται :}$$

$$3\omega^2 - 10\omega + 3 = 0$$

$$\Delta = 64 \quad \omega_{1,2} = \frac{10 \pm 8}{6} \begin{matrix} \nearrow \omega_1 = 3 \\ \searrow \omega_2 = \frac{1}{3} \end{matrix}$$

Άρα: $* \omega = 3 \Rightarrow 9^x = 3 \Rightarrow 9^x = 9^{\frac{1}{2}} \Rightarrow x = \frac{1}{2}$ ή

$* \omega = \frac{1}{3} \Rightarrow 9^x = \frac{1}{3} \Rightarrow 9^x = 9^{-\frac{1}{2}} \Rightarrow x = -\frac{1}{2}$ απορρίπτεται ($x > 0$)

27.7 11)

$$3^{2x+1} + 10 \cdot 3^x + 3 = 0 \Rightarrow 3 \cdot 3^{2x} + 10 \cdot 3^x + 3 = 0$$

θέτω $3^x = \omega$ και η εξίσωση γράφεται :

$$3\omega^2 + 10\omega + 3 = 0$$

$$\Delta = 64 \quad \omega_{1,2} = \frac{-10 \pm 8}{6} \begin{matrix} \nearrow \omega_1 = -3 \\ \searrow \omega_2 = -\frac{1}{3} \end{matrix}$$

Άρα: $* \omega = -3 \Rightarrow 3^x = -3$ αδύνατη ή

$* \omega = -\frac{1}{3} \Rightarrow 3^x = -\frac{1}{3}$ αδύνατη αφού $3^x > 0$ για κάθε $x \in \mathbb{R}$

27.7 12)

$$2^{2+x} - 2^{2-x} = 15 \Rightarrow 2^2 \cdot 2^x - \frac{2^2}{2^x} = 15 \Rightarrow 4 \cdot 2^x - \frac{4}{2^x} = 15$$

θέτω $2^x = \omega$ και η εξίσωση γράφεται :

$$4\omega - \frac{4}{\omega} = 15 \Rightarrow 4\omega^2 - 4 = 15\omega \Rightarrow 4\omega^2 - 15\omega - 4 = 0$$

$$\Delta = 289 \quad \omega_{1,2} = \frac{15 \pm 17}{8} \begin{matrix} \nearrow \omega_1 = -\frac{1}{4} \\ \searrow \omega_2 = 4 \end{matrix}$$

Άρα: $* \omega = -\frac{1}{4} \Rightarrow 2^x = -\frac{1}{4}$ αδύνατη ή

$* \omega = 4 \Rightarrow 2^x = 4 \Rightarrow 2^x = 2^2 \Rightarrow x = 2$

27.7 13)

$$4^x - 2^{x+2} - 32 = 0 \Rightarrow 2^{2x} - 2^2 \cdot 2^x - 32 = 0 \Rightarrow 2^{2x} - 4 \cdot 2^x - 32 = 0$$

θέτω $2^x = \omega$ και η εξίσωση γράφεται :

$$\omega^2 - 4\omega - 32 = 0$$

$$\Delta = 144 \quad \omega_{1,2} = \frac{4 \pm 12}{2} \begin{matrix} \nearrow \omega_1 = 8 \\ \searrow \omega_2 = -4 \end{matrix}$$

Άρα: * $\omega = 8 \Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$ ή

* $\omega = -4 \Rightarrow 2^x = -4$ αδύνατη

27.7 14)

$$9^{x+1} - 27 \cdot 3^x = 0 \Rightarrow 9 \cdot 9^x - 27 \cdot 3^x = 0 \Rightarrow 9 \cdot 3^{2x} - 27 \cdot 3^x = 0$$

θέτω $3^x = \omega$ και η εξίσωση γράφεται :

$$9 \cdot \omega^2 - 27 \cdot \omega = 0 \Rightarrow \omega^2 - 3\omega = 0 \Rightarrow \omega(\omega - 3) = 0 \Rightarrow \omega = 0 \quad \text{ή} \quad \omega = 3$$

Άρα: * $\omega = 0 \Rightarrow 3^x = 0$ αδύνατη ή

* $\omega = 3 \Rightarrow 3^x = 3^1 \Rightarrow x = 1$

27.7 15)

$$4^x - 10 \cdot 2^{x-1} = 24 \Rightarrow 2^{2x} - \frac{10}{2} \cdot 2^x = 24 \Rightarrow 2^{2x} - 5 \cdot 2^x - 24 = 0$$

θέτω $2^x = \omega$ και η εξίσωση γράφεται :

$$\omega^2 - 5\omega - 24 = 0$$

$$\Delta = 121 \quad \omega_{1,2} = \frac{5 \pm 11}{2} \begin{matrix} \nearrow \omega_1 = 8 \\ \searrow \omega_2 = -3 \end{matrix}$$

Άρα: * $\omega = 8 \Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$ ή

* $\omega = -3 \Rightarrow 2^x = -3$ αδύνατη

27.7 16)

$$27^{\frac{x+2}{3}} + 3^{2x+2} = 810 \Rightarrow 3^{\frac{3(x+2)}{3}} + 9 \cdot 3^{2x} = 810 \Rightarrow$$

$$\Rightarrow 3^{x+2} + 9 \cdot 3^{2x} = 810 \Rightarrow 9 \cdot 3^x + 9 \cdot 3^{2x} = 810 \Rightarrow 3^{2x} + 3^x = 90$$

θέτω $3^x = \omega$ και η εξίσωση γράφεται :

$$\omega^2 + \omega - 90 = 0$$

$$\Delta = 361 \quad \omega_{1,2} = \frac{-1 \pm 19}{2} \begin{matrix} \nearrow \omega_1 = 9 \\ \searrow \omega_2 = -10 \end{matrix}$$

Άρα: $* \omega = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2$ ή

$* \omega = -10 \Rightarrow 3^x = -10$ αδύνατη

27.7 17)

$$4^{\frac{2x+3}{2}} - 2^x = \frac{3}{2} \Rightarrow 2^{\frac{2(2x+3)}{2}} - 2^x - \frac{3}{2} = 0 \Rightarrow$$

$$\Rightarrow 2^3 \cdot 2^{2x} - 2^x - \frac{3}{2} = 0 \Rightarrow 8 \cdot 2^{2x} - 2^x - \frac{3}{2} = 0$$

θέτω $2^x = \omega$ και η εξίσωση γράφεται :

$$8\omega^2 - \omega - \frac{3}{2} = 0 \Rightarrow 16\omega^2 - 2\omega - 3 = 0$$

$$\Delta = 196 \quad \omega_{1,2} = \frac{2 \pm 14}{32} \begin{matrix} \nearrow \omega_1 = \frac{1}{2} \\ \searrow \omega_2 = -\frac{3}{8} \end{matrix}$$

Άρα: $* \omega = \frac{1}{2} \Rightarrow 2^x = 2^{-1} \Rightarrow x = -1$ ή

$* \omega = -\frac{3}{8} \Rightarrow 2^x = -\frac{3}{8}$ αδύνατη