27.5 1)

α) $2^{x+1}=4\cdot 8^{3x+5}\Rightarrow 2^{x+1}=2^2\cdot 2^{3(3x+5)}\Rightarrow 2^{x+1}=2^{9x+17}\Rightarrow 9x+17=x+1\Rightarrow 8x=-16\Rightarrow x=-2$

β) $3^x=\sqrt{9^{\frac{4x-3}{x}}}\Rightarrow 3^x=\left(9^{\frac{4x-3}{x}}\right)^{\frac{1}{2}}\Rightarrow 3^x=9^{\frac{4x-3}{2x}}\Rightarrow 3^x=\left(3^2\right)^{\frac{4x-3}{2x}}\Rightarrow 3^x=3^{\frac{2(4x-3)}{2x}}\Rightarrow x=\frac{4x-3}{x}\Rightarrow$

$$\overset{x\neq 0}{\Rightarrow} x^2=4x-3\Rightarrow x^2-4x+3=0\ ,\quad \Delta=4\ ,\quad x_{1,2}=\frac{4\pm 2}{2}\begin{cases}x_1=\dfrac{4+2}{2}=3\\ x_2=\dfrac{4-2}{2}=1\end{cases}$$

27.5 2)

$9^{x+1}=3\cdot 27^x\Rightarrow 3^{2(x+1)}=3\cdot 3^{3x}\Rightarrow 3^{2x+2}=3^{3x+1}\Rightarrow$

$\Rightarrow 2x+2=3x+1\Rightarrow x=1$

27.5 3)

$25^{1+2x}=5\cdot 5^{x-2}\Rightarrow 5^{2(1+2x)}=5^{x-2+1}\Rightarrow 5^{2+4x}=5^{x+1}\Rightarrow$

$\Rightarrow 2+4x=x-1\Rightarrow 3x=-3\Rightarrow x=-1$

27.5 4)

$4^{x-3}=4\cdot 16^{x+2}\Rightarrow 4^{x-3}=4\cdot 4^{2(x+2)}\Rightarrow 4^{x-3}=4^{2x+5}\Rightarrow$

$\Rightarrow x-3=2x+5\Rightarrow x=-8$

27.5 5)

$8^x=4^{x^2+1}\Rightarrow 2^{3x}=2^{2\left(x^2+1\right)}\Rightarrow 3x=2x^2+2\Rightarrow$

$\Rightarrow 2x^2-3x+2=0\quad ,\quad \Delta=-7<0\Rightarrow$ αδύνατη

27.5 6)

$8^{x-1}=4^{2-3x}\Rightarrow 2^{3x-3}=2^{4-6x}\Rightarrow 3x-3=4-6x\Rightarrow 9x=7\Rightarrow x=\frac{7}{9}$

27.5 7)

$16^{2x-1}=\frac{1}{8}\Rightarrow 2^{4(2x-1)}=2^{-3}\Rightarrow 8x-4=4-3\Rightarrow 8x=1\Rightarrow x=\frac{1}{8}$

27.5 8)

$9^{2x^2-5}=27^x\Rightarrow 3^{2\left(2x^2-5\right)}=3^{3x}\Rightarrow 4x^2-10=3x\Rightarrow$

$$\Rightarrow 4x^2-3x-10=0\ ,\quad \Delta=169\quad ,\quad x_{1,2}=\frac{3\pm 13}{8}\begin{cases}x_1=2\\ x_2=-\dfrac{5}{4}\end{cases}$$

27.5 9)

$3^x\cdot 81^3=9^{x+1}\cdot 27^{2x}\Rightarrow 3^x3^{4\cdot 3}=3^{2(x+1)}\cdot 3^{3\cdot 2x}\Rightarrow 3^{x+12}=3^{8x+2}\Rightarrow$

$\Rightarrow x+12=8x+2\Rightarrow 7x=10\Rightarrow x=\frac{10}{7}$

27.5 10)

$$25^{x}\cdot 125^{x+1}=\frac{1}{5^{6}}\Rightarrow 5^{2x}\cdot 5^{3(x+1)}=5^{-6}\Rightarrow 5^{2x+3x+3}=5^{-6}\Rightarrow$$

$$\Rightarrow 5x+3=-6\Rightarrow 5x=-9\Rightarrow x=-\frac{9}{5}$$

27.5 11)

$$4^{2x-3}-16\cdot 2^{x-1}=0\Rightarrow 2^{2(2x-3)}-2^{4}\cdot 2^{x-1}=0\Rightarrow 2^{4x-6}=2^{x+3}\Rightarrow$$

$$\Rightarrow 4x-6=x+3\Rightarrow 3x=9\Rightarrow x=3$$

27.5 12)

$$25^{x+1}-125\cdot 5^{3x-4}=0\Rightarrow 5^{2(x+1)}-5^{3}\cdot 5^{3x-4}=0\Rightarrow 5^{2x+2}=5^{3x-1}\Rightarrow$$

$$\Rightarrow 2x+2=3x-1\Rightarrow x=3$$

27.5 13)

$$2^{\frac{1}{x^{2}}}\cdot 2^{2x+2}=\frac{1}{64}\Rightarrow 2^{-x^{2}}\cdot 2^{2x+2}=2^{-6}\Rightarrow 2^{-x^{2}+2x+2}=2^{-6}\Rightarrow -x^{2}+2x+2=-6\Rightarrow$$

$$\Rightarrow -x^{2}-2x-8=0\ ,\quad \Delta=36\quad ,\quad x_{1,2}=\frac{2\pm 6}{2}\begin{cases}\nearrow x_{1}=4\\ \searrow x_{2}=-2\end{cases}$$

27.5 14)

$$3^{4}\cdot 9^{x^{2}+1}=\frac{3^{x^{2}+3x}}{9^{-x}}\Rightarrow 3^{4}\cdot 3^{2\left(x^{2}+1\right)}=\frac{3^{x^{2}+3x}}{3^{-2x}}\Rightarrow 3^{2x^{2}+6}=3^{x^{2}+3x+2x}\Rightarrow 2x^{2}+6=x^{2}+5x\Rightarrow$$

$$\Rightarrow x^{2}-5x+6=0\ ,\quad \Delta=1\quad ,\quad x_{1,2}=\frac{5\pm 1}{2}\begin{cases}\nearrow x_{1}=3\\ \searrow x_{2}=2\end{cases}$$

27.5 15)

$$\frac{1}{8}\cdot 4^{2x-3}=\left(\frac{\sqrt{2}}{8}\right)^{-x}\Rightarrow 2^{-3}\cdot 2^{2(2x-3)}=\left(\frac{2^{\frac{1}{2}}}{2^{3}}\right)^{-x}\Rightarrow 2^{-3+4x-6}=2^{\left(\frac{1}{2}-3\right)(-x)}\Rightarrow$$

$$\Rightarrow 4x-9=\frac{5x}{2}\Rightarrow 8x-18=5x\Rightarrow 3x=18\Rightarrow x=6$$

27.5 16)

$$\sqrt[x]{64^{2x-1}}=\sqrt{16^{2x-1}}\Rightarrow 2^{\frac{6(2x-1)}{x}}=2^{\frac{4(2x-1)}{2}}\Rightarrow$$

$$\Rightarrow \frac{12x-6}{x}=4x-2\overset{x\neq 0}{\Rightarrow}12x-6=4x^{2}-2x\Rightarrow 4x^{2}-14x+6=0\Rightarrow$$

$$\Rightarrow 2x^{2}-7x+3=0\ ,\quad \Delta=25\quad ,\quad x_{1,2}=\frac{7\pm 5}{4}\begin{cases}\nearrow x_{1}=3\\ \searrow x_{2}=\frac{1}{2}\end{cases}$$