

27.13 1)

$$\alpha) 3^{2x+3} > 3^{x-4} \Rightarrow 2x+3 > x-4 \Rightarrow x > -7$$

$$\beta) \left(\frac{1}{2}\right)^{3-x} \leq \left(\frac{1}{2}\right)^{2x} \stackrel{\frac{1}{2} < 1}{\Rightarrow} 3-x \geq 2x \Rightarrow 3x \leq 3 \Rightarrow x \leq 1$$

$$\gamma) \left(\frac{3}{5}\right)^{2x+4} > \sqrt{\frac{5}{3}} \Rightarrow \left(\frac{3}{5}\right)^{2x+4} > \left(\frac{5}{3}\right)^{\frac{1}{2}} \Rightarrow \left(\frac{3}{5}\right)^{2x+4} > \left(\frac{3}{5}\right)^{-\frac{1}{2}} \stackrel{\frac{3}{5} < 1}{\Rightarrow} 2x+4 > -\frac{1}{2} \Rightarrow 4x+8 > -1 \Rightarrow x > -\frac{9}{4}$$

$$\delta) 25^{x-2} \leq \left(\frac{1}{5}\right)^{x^2-11} \Rightarrow (5^2)^{x-2} \leq (5^{-1})^{x^2-11} \Rightarrow 5^{2x-4} \leq 5^{-x^2+11} \stackrel{5 > 1}{\Rightarrow} 2x-4 \leq -x^2+11 \Rightarrow$$

$$\Rightarrow x^2 + 2x - 15 \leq 0 \quad \Rightarrow \quad \begin{array}{l} \Delta = 4 + 60 = 64, \quad x_{1,2} = \frac{-2 \pm 8}{2} = \begin{cases} x_1 = -5 \\ x_2 = 3 \end{cases} \\ -5 \leq x \leq 3 \end{array}$$

27.13 2)

$$6^{3x-7} \geq 6^{x+5} \Rightarrow 3x-7 \geq x+5 \Rightarrow 2x \geq 12 \Rightarrow x \geq 6$$

27.13 3)

$$\left(\frac{1}{7}\right)^{2x+5} < \left(\frac{1}{7}\right)^{-x+2} \stackrel{\frac{1}{7} < 1}{\Rightarrow} 2x+5 > -x+2 \Rightarrow 3x > -3 \Rightarrow x > -1$$

27.13 4)

$$13^{4x-12} > 1 \Rightarrow 13^{4x-12} > 13^0 \Rightarrow 4x-12 > 0 \Rightarrow 4x > 12 \Rightarrow x > 3$$

27.13 5)

$$\left(\frac{8}{5}\right)^{3x-7} \leq \left(\frac{8}{5}\right)^{x+1} \Rightarrow 3x-7 \leq x+1 \Rightarrow 2x \leq 8 \Rightarrow x \leq 4$$

$$8^{x+1} \leq \left(\frac{1}{32}\right)^{x+9} \Rightarrow (2^3)^{x+1} \leq (2^{-5})^{x+9} \Rightarrow 2^{3x+3} \leq 2^{-5x-45} \Rightarrow$$

$$\Rightarrow 3x+3 \leq -5x-45 \Rightarrow 8x \leq -48 \Rightarrow x \leq -6$$

27.13 12)

$$3^{2-|x|} > 1 \Rightarrow 3^{2-|x|} > 3^0 \Rightarrow 2-|x| > 0 \Rightarrow |x| < 2 \Rightarrow x \in (-2, 2)$$

27.13 13)

$$\left(\frac{4}{3}\right)^{x^2-x} > \left(\frac{4}{3}\right)^{5x+16} \Rightarrow x^2-x > 5x+16 \Rightarrow x^2-6x-16 > 0 \Rightarrow$$

$$\Delta=100, x_{1,2}=\frac{6 \pm 10}{2} \begin{matrix} \nearrow x_1=8 \\ \searrow x_2=-2 \end{matrix}$$

$$\Rightarrow x \in (-\infty, -2) \cup (8, +\infty)$$

27.13 14)

$$\left(\frac{7}{2}\right)^{2x^2+x-15} > 1 \Rightarrow \left(\frac{7}{2}\right)^{2x^2+x-15} > \left(\frac{7}{2}\right)^0 \Rightarrow 2x^2+x-15 > 0 \Rightarrow$$

$$\Delta=121, x_{1,2}=\frac{-1 \pm 11}{4} \begin{matrix} \nearrow x_1=-3 \\ \searrow x_2=\frac{5}{2} \end{matrix}$$

$$\text{Άρα η ανίσωση αληθεύει για } \Rightarrow x \in (-\infty, -3) \cup \left(\frac{5}{2}, +\infty\right)$$

27.13 15)

$$\left(\frac{4}{5}\right)^{2x^2-x-1} \geq 1 \Rightarrow \left(\frac{4}{5}\right)^{2x^2-x-1} \geq \left(\frac{4}{5}\right)^0 \Rightarrow 2x^2-x-1 \leq 0 \Rightarrow$$

$$\Delta=9, x_{1,2}=\frac{1 \pm 3}{4} \begin{matrix} \nearrow x_1=1 \\ \searrow x_2=-\frac{1}{2} \end{matrix}$$

$$\Rightarrow x \in \left[-\frac{1}{2}, 1\right]$$

27.13 16)

Άρα : $x \in (\infty, -2) \cup (-1, 0)$

27.13 18)

$$9^{\frac{1}{x}} \leq \sqrt{3^x} \Rightarrow 3^{\frac{2}{x}} \leq 3^{\frac{x}{2}} \Rightarrow \frac{2}{x} \leq \frac{x}{2} \Rightarrow \frac{2}{x} - \frac{x}{2} \leq 0 \Rightarrow$$

$$\Rightarrow \frac{4}{2x} - \frac{x^2}{2x} \leq 0 \Rightarrow \frac{4-x^2}{2x} \leq 0 \Rightarrow \frac{(2-x)(2+x)}{2x} \leq 0$$

Άρα η ανίσωση αληθεύει για $x \in [-2, 0) \cup [2, +\infty)$

27.13 19)

$$\left(\frac{2}{3}\right)^{x^3+x-\frac{3}{2}} \leq \sqrt{\frac{2}{3}} \Rightarrow \left(\frac{2}{3}\right)^{x^3+x-\frac{3}{2}} \leq \left(\frac{2}{3}\right)^{\frac{1}{2}} \Rightarrow x^3+x-\frac{3}{2} \geq \frac{1}{2} \Rightarrow x^3+x-2 \geq 0$$

Η $x=1$ είναι ρίζα

Σχήμα Horner :

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 & 1 \\ & & 1 & 1 & 2 & \\ \hline & 1 & 1 & 2 & 0 & \end{array}$$

Άρα $x^3+x-2 \geq 0 \Rightarrow (x-1)(x^2+x+2) \geq 0 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$ $x^2+x+2 > 0$ διότι $\Delta = -7 < 0$