

23.2 1)

Θέτουμε $y = x^2 + x - 1$ οπότε η εξίσωση γίνεται

$$(x^2 + x - 1)^3 - (x^2 + x - 2)^2 - 1 = 0 \Rightarrow y^3 - (y - 1)^2 - 1 = 0 \Rightarrow y^3 - (y^2 - 2y + 1) - 1 = 0 \Rightarrow$$

$$\Rightarrow y^3 - y^2 + 2y - 1 - 1 = 0 \Rightarrow y^3 - y^2 + 2y - 2 = 0 \Rightarrow y^2(y - 1) + 2(y - 1) = 0 \Rightarrow (y - 1)(y^2 + 2) = 0 \Rightarrow$$

$$\Rightarrow y - 1 = 0 \Rightarrow y = 1 \xrightarrow{y = x^2 + x - 1} x^2 + x - 1 = 1 \Rightarrow x^2 + x - 2 = 0 \quad \Delta = 4 + 8 = 9, \quad x_{1,2} = \frac{-1 \pm 3}{2} \Rightarrow x = -2 \quad \text{ή} \quad x = 1$$

23.2 2)

$$(2x^2 + 3x - 1)^2 - 5(2x^2 + 3x + 3) + 24 = 0$$

Θέτω $2x^2 + 3x - 1 = \omega$ και η εξίσωση γράφεται :

$$\omega^2 - 5(\omega + 4) + 24 = 0 \Rightarrow \omega^2 - 5\omega + 4 = 0$$

$$\Delta = 25 - 16 = 9, \quad \omega_{1,2} = \frac{5 \pm 3}{2} \Rightarrow \begin{cases} \omega_1 = 4 \\ \omega_2 = 1 \end{cases}$$

Άρα:

$$\omega = 4 \Rightarrow 2x^2 + 3x - 1 = 4 \Rightarrow 2x^2 + 3x - 5 = 0 \quad \Delta = 9 + 40 = 49, \quad x_{1,2} = \frac{-3 \pm 7}{4} \Rightarrow \begin{cases} x_1 = -\frac{5}{2} \\ x_2 = 1 \end{cases}$$

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$$\omega = 1 \Rightarrow 2x^2 + 3x - 1 = 1 \Rightarrow 2x^2 + 3x - 2 = 0 \quad \Delta = 9 + 16 = 25, \quad x_{1,2} = \frac{-3 \pm 5}{4} \Rightarrow \begin{cases} x_1 = -2 \\ x_2 = \frac{1}{2} \end{cases}$$

23.2 3)

$$(x^2 - 2)^3 + 3(x^2 - 3)^2 - 11 = 0$$

Θέτω : $x^2 - 2 = \omega$ και η εξίσωση γράφεται :

$$\omega^3 + 3(\omega - 1)^2 - 11 = 0 \Rightarrow \omega^3 + 3(\omega^2 - 2\omega + 1) - 11 = 0 \Rightarrow \omega^3 + 3\omega^2 - 6\omega - 8 = 0$$

Η $\omega = -1$ είναι ρίζα αφού : $(-1)^3 + 3(-1)^2 - 6(-1) - 8 = 0$

$$\text{Σχήμα Horner : } \begin{array}{r|rrrr} 1 & 3 & -6 & -8 & -1 \\ & -1 & -2 & 8 & \\ \hline 1 & 2 & -8 & 0 & \end{array}$$

$$\text{Άρα: } \omega^3 + 3\omega^2 - 6\omega - 8 = 0 \Rightarrow (\omega + 1)(\omega^2 + 2\omega - 8) = 0 \Rightarrow$$

$$\Rightarrow \omega = -1 \quad \text{ή} \quad \omega^2 + 2\omega - 8 = 0, \quad \Delta = 36, \quad \omega_{1,2} = \frac{-2 \pm 6}{2} \Rightarrow \begin{cases} x_1 = -4 \\ x_2 = 2 \end{cases}$$

$$\text{Άρα: } \omega = -1 \Rightarrow x^2 - 2 = -1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$$\text{ή} \quad \omega = -4 \Rightarrow x^2 - 2 = -4 \Rightarrow x^2 = -2 \quad \text{αδύνατη}$$

$$\text{ή} \quad \omega = 2 \Rightarrow x^2 - 2 = 2 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

23.2 4)

$$(x^2 + x - 5)^3 - 5(x^2 + x - 4)^2 - 7(x^2 + x - 3) + 40 = 0$$

Θέτω : $x^2 + x - 5 = \omega$ και η εξίσωση γράφεται :

$$\omega^3 - 5(\omega + 1)^2 - 7(\omega + 2) + 40 = 0 \Rightarrow \omega^3 - 5(\omega^2 + 2\omega + 1) - 7\omega - 14 + 40 = 0 \Rightarrow$$

$$\Rightarrow \omega^3 - 5\omega^2 - 10\omega - 5 - 7\omega + 26 = 0 \Rightarrow \omega^3 - 5\omega^2 - 17\omega + 21 = 0$$

Η $\omega = 1$ είναι ρίζα αφού : $1^3 - 5 \cdot 1^2 - 17 \cdot 1 + 21 = -21 + 21 = 0$

$$\text{Σχήμα Horner : } \begin{array}{r|rrrr} 1 & -5 & -17 & 21 & 1 \\ & 1 & -4 & -21 & \\ \hline 1 & -4 & -21 & 0 & \end{array}$$

$$\text{Άρα: } \omega^3 - 5\omega^2 - 17\omega + 21 = 0 \Rightarrow (\omega - 1)(\omega^2 - 4\omega - 21) = 0 \Rightarrow$$

$$\Rightarrow \omega = 1 \quad \text{ή} \quad \omega^2 - 4\omega - 21 = 0, \Delta = 100, \omega_{1,2} = \frac{4 \pm 10}{2} \Rightarrow \begin{cases} \omega_1 = 7 \\ \omega_2 = -3 \end{cases}$$

$$\text{Άρα: } \omega = 1 \Rightarrow x^2 + x - 5 = 1 \Rightarrow x^2 + x - 6 = 0 \quad \Delta = 25, \quad x_{1,2} = \frac{-1 \pm 5}{2} \Rightarrow \begin{cases} x_1 = -3 \\ x_2 = 2 \end{cases}$$

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$$\omega = 7 \Rightarrow x^2 + x - 5 = 7 \Rightarrow x^2 + x - 12 = 0 \quad \Delta = 49, \quad x_{1,2} = \frac{-1 \pm 7}{2} \Rightarrow \begin{cases} x_1 = -4 \\ x_2 = 3 \end{cases}$$

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$$\omega = -3 \Rightarrow x^2 + x - 5 = -3 \Rightarrow x^2 + x - 2 = 0 \quad \Delta = 9, \quad x_{1,2} = \frac{-1 \pm 3}{2} \Rightarrow \begin{cases} x_1 = -2 \\ x_2 = 1 \end{cases}$$

23.2 5)

$$(x^2 + 2x - 2)^3 - 4(x^2 + 2x - 4)^2 - 23(x^2 + 2x - 4) - 24 = 0$$

Θέτω : $x^2 + 2x - 2 = \omega$ και η εξίσωση γράφεται :

$$\omega^3 - 4(\omega - 1)^2 - 23(\omega - 2) - 24 = 0 \Rightarrow \omega^3 - 4(\omega^2 - 2\omega + 1) - 23\omega + 46 - 24 = 0 \Rightarrow$$

$$\Rightarrow \omega^3 - 4\omega^2 + 8\omega - 4 - 23\omega + 22 = 0 \Rightarrow \omega^3 - 4\omega^2 - 15\omega + 18 = 0$$

Η $\omega = 1$ είναι ρίζα αφού : $1^3 - 4 \cdot 1^2 - 15 \cdot 1 + 18 = -18 + 18 = 0$

$$\text{Σχήμα Horner : } \begin{array}{r|rrrr} 1 & -4 & -15 & 18 & 1 \\ & 1 & -3 & -18 & \\ \hline 1 & -3 & -18 & 0 & \end{array}$$

$$\text{Άρα: } \omega^3 - 4\omega^2 - 15\omega + 18 = 0 \Rightarrow (\omega - 1)(\omega^2 - 3\omega - 18) = 0 \Rightarrow$$

$$\Rightarrow \omega = 1 \quad \text{ή} \quad \omega^2 - 3\omega - 18 = 0, \Delta = 9 + 72 = 81, \omega_{1,2} = \frac{3 \pm 9}{2} \Rightarrow \begin{cases} \omega_1 = 6 \\ \omega_2 = -3 \end{cases}$$

$$\text{Άρα: } \omega = 1 \Rightarrow x^2 + 2x - 2 = 1 \Rightarrow x^2 + 2x - 3 = 0 \quad \Delta = 16, \quad x_{1,2} = \frac{-2 \pm 4}{2} \Rightarrow \begin{cases} x_1 = \boxed{-3} \\ x_2 = \boxed{1} \end{cases}$$

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$$\omega = 6 \Rightarrow x^2 + 2x - 2 = 6 \Rightarrow x^2 + 2x - 8 = 0 \quad \Delta = 36, \quad x_{1,2} = \frac{-2 \pm 6}{2} \Rightarrow \begin{matrix} \nearrow x_1 = \boxed{-4} \\ \searrow x_2 = \boxed{2} \end{matrix}$$

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$$\omega = -3 \Rightarrow x^2 + 2x - 2 = -3 \Rightarrow x^2 + 2x + 1 = 0 \Rightarrow (x+1)^2 = 0 \Rightarrow x+1 = 0 \Rightarrow x = \boxed{-1}$$