20.1 1)

$$\begin{array}{r|l} 6x^3-19x^2+20x-10 & \underline{3x^2-5x+6} \\ \underline{-\left(6x^3-10x^2+12x\right)} & 2x-3 \\ -9x^2+8x-10 & \\ \underline{-\left(9x^2+15x-18\right)} & \\ \end{array}$$

Άρα $\left(6x^3-19x^2+20x-10\right)=\left(3x^2-5x+6\right)(2x-3)-7x+8$

Επαλήθευση : $\left(3x^2-5x+6\right)(2x-3)-7x+8=6x^3-9x^2-10x^2+15x+12x-18-7x+8=$

$=6x^3-19x^2+20x-10$

20.1 2)

$$\begin{array}{r|l} 3x^3-x^2+4x-3 & \underline{x^2+x+2} \\ \underline{-3x^3-3x^2-6x} & 3x-4 \\ -4x^2-2x-3 & \\ \underline{4x^2+4x+8} & \\ 2x+5 & \end{array}$$

Επομένως $3x^3-x^2+4x-3=\left(x^2+x+2\right)(3x-4)+2x+5$

Επαλήθευση $\left(x^2+x+2\right)(3x-4)+2x+5=$

$=3x^3-4x^2+3x^2-4x+6x-8+2x+5=3x^3-x^2+4x-3$

20.1 3)

$\left(6x^3-13x^2+7x+5\right):\left(2x^2-3x+4\right)$

$$\begin{array}{r|l} 6x^3-13x^2+7x+5 & \underline{2x^2-3x+4} \\ \underline{-\left(6x^3-9x^2+12x\right)} & 3x-2 \\ -4x^2-5x+5 & \\ \underline{-\left(-4x^2+6x-8\right)} & \\ -11x+13 & \end{array}$$

Άρα $\left(2x^2-3x+4\right)(3x-2)-11x+13=6x^3-13x^2+7x+5$

Επαλήθευση : $\left(2x^2-3x+4\right)(3x-2)-11x+13=6x^3-9x^2+12x-4x^2+6x+12x-8-11x+13=$

$=6x^3-13x^2+7x+5$

20.1 4)

$\left(x^3-x^2-9x-10\right):\left(x-4\right)$

$$\begin{array}{r|l} x^3-x^2-9x-10 & x-4 \\ \hline -(x^3-4x^2) & x^2+3x+3 \\ \hline 3x^2-9x-10 & \\ -(3x^2-12x) & \\ \hline 3x-10 & \\ -(3x-12) & \\ \hline 2 & \end{array}$$

Άρα $(x^3-x^2-9x-10)=(x-4)(x^2+3x+3)+2$

Επαλήθευση : $(x-4)(x^2+3x+3)+2=x^3+3x^2+3x-4x^2-12x-12+2=$

$=x^3-x^2-9x-10$

20.1 5)

$(6x^3-x^2-14x+3):(2x-3)$

$$\begin{array}{r|l} 6x^3-x^2-14x+3 & 2x-3 \\ \hline -(6x^3-9x^2) & 3x^2+4x-1 \\ \hline 8x^2-14x+3 & \\ -(8x^2-12x) & \\ \hline -2x+3 & \\ -(-2x+3) & \\ \hline 0 & \end{array}$$

Άρα $6x^3-x^2-14x+3=(2x-3)(3x^2+4x-1)$

Επαλήθευση : $(2x-3)(3x^2+4x-1)=6x^3-8x^2-2x-9x^2-12x+3=$

$=6x^3-x^2-14x+3$

20.1 6)

$(3x^3+2x^2+1):(x^2+x-2)$

$$\begin{array}{r|l} 3x^3+2x^2+1 & x^2+x-2 \\ \hline -(3x^3+3x^2-6x) & 3x-1 \\ \hline -x^2+6x+1 & \\ -(x^2-x+2) & \\ \hline 7x-1 & \end{array}$$

Άρα $3x^3+2x^2+1=(x^2+x-2)(3x-1)+7x-1$

Επαλήθευση : $(x^2+x-2)(3x-1)+7x-1=3x^3-x^2+3x^2-x-6x+2+7x-1$

$=3x^3+2x^2+1$

20.1 7)

$(x^4+x^3+7x^2-6x+8):(x^2-x+1)$

$$\begin{array}{r|l} x^4+x^3+7x^2-6x+8 & \underline{x^2-x+1} \\ -\left(x^4-x^3+x^2\right) & x^2+2x+8 \\ \hline 2x^3+6x^2-6x+8 & \\ -\left(2x^3-2x^2+2x\right) & \\ \hline 8x^2-8x+8 & \\ -\left(8x^2-8x+8\right) & \\ \hline 0 & \end{array}$$

Άρα $\left(x^4+x^3-7x^2-6x+8\right)=\left(x^2-x+1\right)\left(x^2+2x+8\right)$

Επαλήθευση: $\left(x^2-x+1\right)\left(x^2+2x+8\right)=x^4+2x^3+8x^2-x^3-2x^2-8x+x^2+2x+8=$

$=x^4+x^3-7x^2-6x+8$

20.1 8)

$\left(2x^4-6x^3+3x^2-3x+1\right):\left(x^2-3x+1\right)$

$$\begin{array}{r|l} 2x^4-6x^3+3x^2-3x+1 & \underline{x^2-3x+1} \\ -\left(2x^2-6x^3+2x^2\right) & 2x^2+1 \\ \hline x^2-3x+1 & \\ -\left(x^2-3x+1\right) & \\ \hline 0 & \end{array}$$

Άρα $\left(2x^4-6x^3+3x^2-3x+1\right)=\left(x^2-3x+1\right)\left(2x^2+1\right)$

Επαλήθευση: $\left(x^2-3x+1\right)\left(2x^2+1\right)=2x^4-6x^3+2x^2+x^2-3x+1=$

$=2x^4-6x^3+3x^2-3x+1$

20.1 9)

$\left(x^4+x^3-7x^2+3x-6\right):\left(x-2\right)$

$$\begin{array}{r|l} x^4+x^3-7x^2+3x-6 & \underline{x-2} \\ -\left(x^4-2x^3\right) & x^3+3x^2-x+1 \\ \hline 3x^3-7x^2+3x-6 & \\ -\left(3x^3-6x^2\right) & \\ \hline -x^2+3x-6 & \\ -\left(-x^2+2x\right) & \\ \hline x-6 & \\ -\left(x-2\right) & \\ \hline 4 & \end{array}$$

Άρα $x^4+x^3-7x^2+3x-6=\left(x-2\right)\left(x^3+3x^2-x+1\right)-4$

Επαλήθευση: $\left(x-2\right)\left(x^3+3x^2-x+1\right)-4=x^4+3x^3-x^2+x-2x^3-6x^2+2x-2-4=$

$=x^4+x^3-7x^2+3x-6$

20.1 10)

$$\left(2x^4-3x^3+5x^2-3\right):\left(2x-1\right)$$

$$\begin{array}{r|l}
2x^4-3x^3+5x^2-3 & \underline{2x-1} \\
-\left(2x^4-x^3\right) & x^3-x^2+2x+1 \\
\hline
-2x^3+5x^2-3 & \\
-\left(-2x^3+x^2\right) & \\
\hline
4x^2-3 & \\
-\left(4x^2-2x\right) & \\
\hline
2x-3 & \\
-\left(2x-1\right) & \\
\hline
-2 &
\end{array}$$

Άρα $2x^4-3x^3+5x^2-3=\left(2x-1\right)\left(x^3-x^2+2x+1\right)-2$

Επαλήθευση: $\left(2x-1\right)\left(x^3-x^2+2x+1\right)-2=2x^4-2x^3+4x^2+2x-x^3+x^2-2x-1-2=$

$=2x^2-3x^3+5x^2-3$

20.1 11)

$$\left(x^4-x^2+1\right):\left(x^2+x+1\right)$$

$$\begin{array}{r|l}
x^4-x^2+1 & \underline{x^2+x+1} \\
-\left(x^4+x^3+x^2\right) & x^2-x-1 \\
\hline
-x^3-2x^2+1 & \\
-\left(-x^3-x^2-x\right) & \\
\hline
-x^2+x+1 & \\
-\left(-x^2-x-1\right) & \\
\hline
2x+2 &
\end{array}$$

Άρα $\left(x^4-x^2+1\right)=\left(x^2+x+1\right)\left(x^2-x-1\right)+2x+2$

Επαλήθευση: $\left(x^2+x+1\right)\left(x^2-x-1\right)+2x+2=x^4-x^3-x^2+x^3-x^2-x+x^2-x-1+2x+2=$

$=x^4-x^2+1$

20.1 12)

$$\begin{array}{r|l}
x^5-6x^3+3x^2+7x-4 & \underline{x^3+x^2-2x+1} \\
-5x^5-x^4+2x^3-x^2 & x^2-x-3 \\
\hline
-x^4-4x^3+2x^2+7x-4 & \\
x^4+x^3-2x^2+x & \\
\hline
-3x^3+8x-4 & \\
3x^3+3x^2-6x+3 & \\
\hline
3x^2+2x-1 &
\end{array}$$

Επομένως $x^5-6x^3+3x^2+7x-4=\left(x^3+x^2-2x+1\right)\left(x^2-x-3\right)+3x^2+2x-1$

Επαλήθευση $\left(x^3+x^2-2x+1\right)\left(x^2-x-3\right)+3x^2+2x-1=$

$=x^5\cancel{-x^4}-3x^3\cancel{+x^4}-x^3\cancel{-3x^2}-2x^3+2x^2+6x+x^2-x-3\cancel{+3x^2}+2x-1=$

$= x^5 - 6x^3 + 3x^2 + 7x - 4$

20.1 13)

$\left(x^3+1\right):\left(x+1\right)$

$$\begin{array}{r|l}
x^3+1 & \underline{x+1} \\
-\left(x^3+x^2\right) & x^2-x+1 \\
\hline
x^2+1 & \\
-\left(-x^2-x\right) & \\
\hline
x+1 & \\
-\left(x+1\right) & \\
\hline
0 &
\end{array}$$

Άρα $x^3+1=\left(x+1\right)\left(x^2-x+1\right)$

Επαλήθευση : $\left(x+1\right)\left(x^2-x+1\right)=x^3-x^2+x+x^2-x+1=x^3+1$

20.1 14)

$x^5:\left(x-1\right)^2$ είναι $\left(x-1\right)^2=x^2-2x+1$

$$\begin{array}{r|l}
x^5 & \underline{x^2-2x+1} \\
-\left(x^5-2x^4+x^3\right) & x^3+2x^2+3x+4 \\
\hline
2x^4-x^3 & \\
-\left(2x^4-4x^3+2x^2\right) & \\
\hline
3x^3-2x^2 & \\
-\left(3x^3-6x^2+3x\right) & \\
\hline
4x^2-3x & \\
-\left(4x^2-8x+4\right) & \\
\hline
5x-4 &
\end{array}$$

Άρα $x^5=\left(x^2-2x+1\right)\left(x^3+2x^2+3x+4\right)+5x-4$

Επαλήθευση : $\left(x^2-2x+1\right)\left(x^3+2x^2+3x+4\right)+5x-4=$

$= x^5+2x^4+3x^3+4x^2-2x^4-4x^3-6x^2-4x+x^3+2x^2+3x+4+5x-4=x^5$