

19.9 1)

$$\text{Έστω } \frac{5x+7}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

(Σπάμε το κλάσμα σε άθροισμα δύο κλασμάτων, με παρονομαστή πολυώνυμο πρώτου βαθμού και αριθμητή σταθερά, ώστε αφού γίνουν οι πράξεις στο δεξί μέλος, να προκύψει στον αριθμητή πολυώνυμο πρώτου βαθμού).

$$\text{Άρα έχουμε : } \frac{5x+7}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \Rightarrow$$

$$\Rightarrow \frac{5x+7}{(x+1)(x+2)} = \frac{A(x+2)}{(x+1)(x+2)} + \frac{B(x+1)}{(x+1)(x+2)} \Rightarrow$$

$$\Rightarrow \frac{5x+7}{(x+1)(x+2)} = \frac{Ax+2A+Bx+1}{(x+1)(x+2)} \Rightarrow \frac{5x+7}{(x+1)(x+2)} = \frac{(A+B)x+(2A+1)}{(x+1)(x+2)} \Rightarrow$$

$$\Rightarrow 5x+7 = (A+B)x + (2A+1) \xrightarrow{\text{ισότητα πολυωνύμων}} \begin{cases} 5 = A+B \\ 7 = 2A+1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} B = 5-A \\ 2A = 6 \end{cases} \Rightarrow \begin{cases} B = 2 \\ A = 3 \end{cases} \quad \text{άρα} \quad \frac{5x+7}{(x+1)(x+2)} = \frac{3}{x+1} + \frac{2}{x+2}$$

19.9 2)

Παραγοντοποιούμε το τριώνυμο $x^2 - 4x + 3$

$$\Delta = (-4)^2 - 4 \cdot 3 = 4 \quad x_{1,2} = \frac{4 \pm 2}{2} \Rightarrow \begin{cases} x_1 = 3 \\ x_2 = 1 \end{cases}$$

$$\text{Άρα : } x^2 - 4x + 3 = (x-3)(x-1)$$

$$\text{Έστω : } \frac{3x+1}{x^2-4x+3} = \frac{3x+1}{(x-3)(x-1)} = \frac{A}{(x-3)} + \frac{B}{(x-1)} \Rightarrow$$

$$\Rightarrow \frac{(3x+1)}{(x-3)(x-1)} = \frac{A(x-1)}{(x-3)(x-1)} + \frac{B(x-3)}{(x-3)(x-1)} \Rightarrow$$

$$\Rightarrow \frac{3x+1}{(x-3)(x-1)} = \frac{Ax-A+Bx-3B}{(x-3)(x-1)} \Rightarrow \frac{3x+1}{(x-3)(x-1)} = \frac{(A+B)x-A-3B}{(x-3)(x-1)} \Rightarrow$$

$$\Rightarrow 3x+1 = (A+B)x - A - 3B \xrightarrow{\text{ισότητα πολυωνύμων}} \begin{cases} A+B=3 \\ -A-3B=1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} A=3-B \\ -(3-B)-3B=1 \end{cases} \Rightarrow \begin{cases} A=3-B \\ -2B=4 \end{cases} \Rightarrow \begin{cases} A=3-B \\ B=-2 \end{cases} \Rightarrow \begin{cases} A=5 \\ B=-2 \end{cases}$$

$$\text{Άρα : } \frac{3x+1}{x^2-4x+3} = \frac{5}{x-3} + \frac{-2}{x-1}$$

19.9 3)

Παραγοντοποιούμε το τριώνυμο $x^2 - 2x - 15$

$$\Delta = 2^2 + 4 \cdot 15 = 64 \quad x_{1,2} = \frac{2 \pm 8}{2} \Rightarrow \begin{cases} x_1 = 5 \\ x_2 = -3 \end{cases}$$

$$\text{Άρα: } x^2 - 2x - 15 = (x - 5)(x + 3)$$

$$\text{Άρα: } \frac{6x + 2}{x^2 - 2x - 15} = \frac{6x + 2}{(x - 5)(x + 3)}$$

$$\text{Έστω: } \frac{6x + 2}{(x - 5)(x + 3)} = \frac{A}{x - 5} + \frac{B}{x + 3} \Rightarrow$$

$$\Rightarrow \frac{6x + 2}{(x - 5)(x + 3)} = \frac{A(x + 3)}{(x - 5)(x + 3)} + \frac{B(x - 5)}{(x - 5)(x + 3)} \Rightarrow$$

$$\Rightarrow \frac{6x + 2}{(x - 5)(x + 3)} = \frac{Ax + 3A + Bx - 5B}{(x - 5)(x + 3)} \Rightarrow \frac{6x + 2}{(x - 5)(x + 3)} = \frac{(A + B)x + 3A - 5B}{(x - 5)(x + 3)} \Rightarrow$$

$$\Rightarrow 6x + 2 = (A + B)x + 3A - 5B \xrightarrow{\text{ισότητα πολυωνύμων}} \begin{cases} A + B = 6 \\ 3A - 5B = 2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} A = 6 - B \\ 3(6 - B) - 5B = 2 \end{cases} \Rightarrow \begin{cases} A = 6 - B \\ 18 - 3B - 5B = 2 \end{cases} \Rightarrow \begin{cases} A = 6 - B \\ -8B = -16 \end{cases} \Rightarrow \begin{cases} A = 4 \\ B = 2 \end{cases}$$

$$\text{Άρα: } \frac{6x + 2}{x^2 - 2x - 15} = \frac{4}{x - 5} + \frac{2}{x + 3}$$

19.9 4)

Παραγοντοποιούμε το τριώνυμο $2x^2 - 5x + 2$

$$\Delta = 25 - 4 \cdot 2 \cdot 2 = 9 \quad x_{1,2} = \frac{5 \pm 3}{4} \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = \frac{1}{2} \end{cases}$$

$$\text{Άρα: } 2x^2 - 5x + 2 = (x - 2)(2x - 1) \quad \text{και} \quad \frac{x - 11}{2x^2 - 5x + 2} = \frac{x - 11}{(x - 2)(2x - 1)}$$

$$\text{Έστω: } \frac{x - 11}{(x - 2)(2x - 1)} = \frac{A}{x - 2} + \frac{B}{2x - 1} \Rightarrow$$

$$\Rightarrow \frac{x - 11}{(x - 2)(2x - 1)} = \frac{A(2x - 1)}{(x - 2)(2x - 1)} + \frac{B(x - 2)}{(x - 2)(2x - 1)} \Rightarrow$$

$$\Rightarrow \frac{x - 11}{(x - 2)(2x - 1)} = \frac{2Ax - A + Bx - 2B}{(x - 2)(2x - 1)} \Rightarrow \frac{x - 11}{(x - 2)(2x - 1)} = \frac{(2A + B)x - A - 2B}{(x - 2)(2x - 1)} \Rightarrow$$

$$\Rightarrow x - 11 = (2A + B)x - A - 2B \xrightarrow{\text{ισότητα πολυωνύμων}} \begin{cases} 2A + B = 1 \\ -A - 2B = -11 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} B = 1 - 2A \\ -A - 2B = -11 \end{cases} \Rightarrow \begin{cases} B = 1 - 2A \\ -A - 2(1 - 2A) = -11 \end{cases} \Rightarrow \begin{cases} B = 1 - 2A \\ -A - 2 + 4A = -11 \end{cases} \Rightarrow \begin{cases} B = 1 - 2A \\ 3A = -9 \end{cases} \Rightarrow \begin{cases} B = 7 \\ A = -3 \end{cases}$$

$$\text{Apr}\alpha: \frac{x-11}{2x^2-5x+2} = \frac{7}{2x-1} - \frac{3}{x-2}$$