

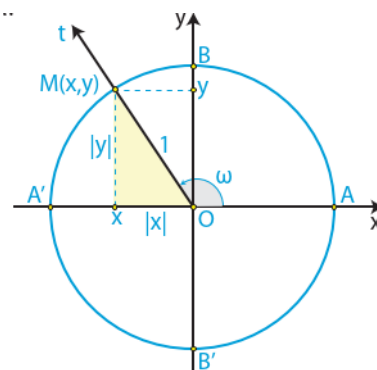
ΘΕΜΑ 1

A. α) Σ β) Λ γ) Σ δ) Σ ε) Σ

B. Στο ίδιο σχήμα έχουμε:

$$\varepsilon\varphi\omega = \frac{y}{x} = \frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} \quad (\text{εφόσον } x = \sigma\upsilon\nu\omega \neq 0)$$

$$\sigma\varphi\omega = \frac{x}{y} = \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega} \quad (\text{εφόσον } y = \eta\mu\omega \neq 0)$$



ΘΕΜΑ 2

α) Το συνημίτονο της ω είναι η τετμημένη του σημείου B του τριγωνομετρικού κύκλου. Άρα $\sigma\upsilon\nu\omega = \frac{-3}{5}$

β) Η γωνία β βρίσκεται στο 2° τεταρτημόριο, άρα θα είναι $\eta\mu\omega > 0$

$$\text{Έχουμε: } \eta\mu^2\omega + \sigma\upsilon\nu^2\omega = 1 \Rightarrow \eta\mu^2\omega + \left(-\frac{3}{5}\right)^2 = 1 \Rightarrow$$

$$\Rightarrow \eta\mu^2\omega + \frac{9}{25} = 1 \Rightarrow \eta\mu^2\omega = \frac{16}{25} \Rightarrow \eta\mu\omega = \pm \frac{4}{5} \xrightarrow{\eta\mu\omega > 0} \eta\mu\omega = \frac{4}{5}$$

ΘΕΜΑ 3

$$\eta\mu^2\left(2x - \frac{\pi}{4}\right) - \eta\mu^2\left(x + \frac{\pi}{4}\right) = 0 \Rightarrow$$

$$\Rightarrow \left(\eta\mu\left(2x - \frac{\pi}{4}\right) - \eta\mu\left(x + \frac{\pi}{4}\right)\right) \cdot \left(\eta\mu\left(2x - \frac{\pi}{4}\right) + \eta\mu\left(x + \frac{\pi}{4}\right)\right) = 0 \Rightarrow$$

$$\Rightarrow \eta\mu\left(2x - \frac{\pi}{4}\right) = \eta\mu\left(x + \frac{\pi}{4}\right) \quad \text{ή} \quad \eta\mu\left(2x - \frac{\pi}{4}\right) = -\eta\mu\left(x + \frac{\pi}{4}\right) \Rightarrow$$

$$\Rightarrow \eta\mu\left(2x - \frac{\pi}{4}\right) = \eta\mu\left(x + \frac{\pi}{4}\right) \quad \text{ή} \quad \eta\mu\left(2x - \frac{\pi}{4}\right) = \eta\mu\left(-x - \frac{\pi}{4}\right) \Rightarrow$$

$$\Rightarrow 2x - \frac{\pi}{4} = 2k\pi + x + \frac{\pi}{4} \quad \text{ή} \quad 2x - \frac{\pi}{4} = 2k\pi + \pi - x - \frac{\pi}{4}$$

$$\text{ή} \quad 2x - \frac{\pi}{4} = 2k\pi - x - \frac{\pi}{4} \quad \text{ή} \quad 2x - \frac{\pi}{4} = 2k\pi + \pi + x + \frac{\pi}{4} \Rightarrow$$

$$\Rightarrow x = 2k\pi + \frac{\pi}{2} \quad \text{ή} \quad 3x = 2k\pi + \pi \quad \text{ή} \quad 3x = 2k\pi \quad \text{ή} \quad x = 2k\pi + \frac{3\pi}{2} \Rightarrow$$

$$\Rightarrow x = 2k\pi + \frac{\pi}{2} \quad \text{ή} \quad x = \frac{2k\pi}{3} + \frac{\pi}{3} \quad \text{ή} \quad x = \frac{2k\pi}{3} \quad \text{ή} \quad x = 2k\pi + \frac{3\pi}{2}, \quad k \in \mathbb{Z}$$

ΘΕΜΑ 4

α) Έχουμε από εκφώνηση: $\eta\mu\omega + \sigma\upsilon\nu\omega = \lambda \Rightarrow$

$$\Rightarrow (\eta\mu\omega + \sigma\upsilon\nu\omega)^2 = \lambda^2 \Rightarrow \eta\mu^2\omega + 2\eta\mu\omega \cdot \sigma\upsilon\nu\omega + \sigma\upsilon\nu^2\omega = \lambda^2 \Rightarrow$$

$$\Rightarrow 1 + 2\eta\mu\omega \cdot \sigma\upsilon\nu\omega = \lambda^2 \Rightarrow 2\eta\mu\omega \cdot \sigma\upsilon\nu\omega = \lambda^2 - 1 \Rightarrow \eta\mu\omega \cdot \sigma\upsilon\nu\omega = \frac{\lambda^2 - 1}{2}$$

β) Θα υπολογίσουμε αρχικά την παράσταση $\eta\mu^4\omega + \sigma\upsilon\nu^4\omega$

$$\text{Έχουμε ότι: } \eta\mu^2\omega + \sigma\upsilon\nu^2\omega = 1 \Rightarrow (\eta\mu^2\omega + \sigma\upsilon\nu^2\omega)^2 = 1 \Rightarrow$$

$$\begin{aligned}
&\Rightarrow \eta\mu^4\omega + 2\eta\mu^2\omega \cdot \sigma\upsilon\nu^2\omega + \sigma\upsilon\nu^4\omega = 1 \Rightarrow \\
&\Rightarrow \eta\mu^4\omega + \sigma\upsilon\nu^4\omega = 1 - 2\eta\mu^2\omega \cdot \sigma\upsilon\nu^2\omega \Rightarrow \\
&\Rightarrow \eta\mu^4\omega + \sigma\upsilon\nu^4\omega = 1 - 2 \cdot \left(\frac{\lambda^2 - 1}{2}\right)^2 \Rightarrow \\
&\Rightarrow \eta\mu^4\omega + \sigma\upsilon\nu^4\omega = 1 - 2 \cdot \frac{(\lambda^4 - 2\lambda + 1)}{4} \Rightarrow \\
&\Rightarrow \eta\mu^4\omega + \sigma\upsilon\nu^4\omega = \frac{1 - \lambda^4 + 2\lambda^2}{2}
\end{aligned}$$

$$\begin{aligned}
&\text{Επίσης είναι : } \eta\mu^6\omega + \sigma\upsilon\nu^6\omega = (\eta\mu^2\omega)^3 + (\sigma\upsilon\nu^2\omega)^3 = \\
&= (\eta\mu^2\omega + \sigma\upsilon\nu^2\omega) \left((\eta\mu^2\omega)^2 - \eta\mu^2\omega \cdot \sigma\upsilon\nu^2\omega + (\sigma\upsilon\nu^2\omega)^2 \right) = \\
&= 1 \cdot (\eta\mu^4\omega - \eta\mu^2\omega \cdot \sigma\upsilon\nu^2\omega + \sigma\upsilon\nu^4\omega) = \\
&= \frac{1 - \lambda^4 + 2\lambda^2}{2} - \left(\frac{\lambda^2 - 1}{2}\right)^2 = \\
&= \frac{1 - \lambda^4 + 2\lambda^2}{2} - \frac{(\lambda^4 - 2\lambda^2 + 1)}{4} = \frac{1 - 3\lambda^4 + 6\lambda^2}{4}
\end{aligned}$$