

Β ΛΥΚΕΙΟΥ ΑΛΓΕΒΡΑ

17.36

Αρχικά χρησιμοποιούμε την ταυτότητα $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

με $\alpha = \eta\mu^2\omega$ και $\beta = \sigma v^2\omega$ και έχουμε :

$$\begin{aligned}\eta\mu^6\omega + \sigma v^6\omega &= (\eta\mu^2\omega)^3 + (\sigma v^2\omega)^3 = \\ &= (\eta\mu^2\omega + \sigma v^2\omega)(\eta\mu^4\omega - \eta\mu^2\omega \cdot \sigma v^2\omega + \sigma v^4\omega) = \\ &= 1 \cdot (\eta\mu^4\omega - \eta\mu^2\omega \cdot \sigma v^2\omega + \sigma v^4\omega) = \\ &= \eta\mu^4\omega + \sigma v^4\omega - \eta\mu^2\omega \cdot \sigma v^2\omega = \\ &= \left[(\eta\mu^2\omega + \sigma v^2\omega)^2 - 2\eta\mu^2\omega \cdot \sigma v^2\omega \right] - \eta\mu^2\omega \cdot \sigma v^2\omega = \\ &= 1 - 3\eta\mu^2\omega \cdot \sigma v^2\omega\end{aligned}$$

Οπότε $\eta\mu^6\omega + \sigma v^6\omega = 1 - 3\eta\mu^2\omega \cdot \sigma v^2\omega$

Οπότε τώρα έχουμε

$$\begin{aligned}\alpha' \text{ μέλος} &= \eta\mu^6\theta + \sigma v^6\theta - 2\eta\mu^4\theta - \sigma v^4\theta + \eta\mu^2\theta = \\ &= 1 - 3\eta\mu^2\theta \cdot \sigma v^2\theta - 2\eta\mu^4\theta - \sigma v^4\theta + \eta\mu^2\theta = \\ &= 1 - 2\eta\mu^2\theta \cdot \sigma v^2\theta - \eta\mu^2\theta \cdot \sigma v^2\theta - 2\eta\mu^4\theta - \sigma v^4\theta + \eta\mu^2\theta = \\ &= 1 - 2\eta\mu^2\theta(\sigma v^2\theta + \eta\mu^2\theta) - \sigma v^2\theta(\eta\mu^2\theta + \sigma v^2\theta) + \eta\mu^2\theta = \\ &= 1 - 2\eta\mu^2\theta - \sigma v^2\theta + \eta\mu^2\theta = 1 - (\eta\mu^2\theta + \sigma v^2\theta) = \\ &= 1 - 1 = 0 = \\ &= \beta' \text{ μέλος}\end{aligned}$$