

$$\eta\mu^2 2x - \sigma\upsilon\nu^2\left(x - \frac{\pi}{3}\right) = 0 \Rightarrow \sigma\upsilon\nu^2\left(\frac{\pi}{2} - 2x\right) - \sigma\upsilon\nu^2\left(x - \frac{\pi}{3}\right) = 0$$

$$\Rightarrow \sigma\upsilon\nu^2\left(\frac{\pi}{2} - 2x\right) = \sigma\upsilon\nu^2\left(x - \frac{\pi}{3}\right) \Rightarrow$$

$$\sigma\upsilon\nu\left(\frac{\pi}{2} - 2x\right) = \sigma\upsilon\nu\left(x - \frac{\pi}{3}\right) \Rightarrow$$

$$\frac{\pi}{2} - 2x = 2\kappa\pi + x - \frac{\pi}{3}$$

$$\Rightarrow -3x = 2\kappa\pi - \frac{\pi}{3} - \frac{\pi}{2}$$

$$\Rightarrow -3x = 2\kappa\pi - \frac{5\pi}{6}$$

$$\Rightarrow x = \frac{5\pi}{18} - \frac{2\kappa\pi}{3}$$

$$\frac{\pi}{2} - 2x = 2\kappa\pi - x + \frac{\pi}{3}$$

$$\Rightarrow -x = 2\kappa\pi + \frac{\pi}{3} - \frac{\pi}{2}$$

$$\Rightarrow -x = 2\kappa\pi - \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6} - 2\kappa\pi$$

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$$\sigma\upsilon\nu\left(\frac{\pi}{2} - 2x\right) = -\sigma\upsilon\nu\left(x - \frac{\pi}{3}\right) \Rightarrow$$

$$\Rightarrow \sigma\upsilon\nu\left(\frac{\pi}{2} - 2x\right) = \sigma\upsilon\nu\left(\pi - x + \frac{\pi}{3}\right) \Rightarrow$$

$$\Rightarrow \sigma\upsilon\nu\left(\frac{\pi}{2} - 2x\right) = \sigma\upsilon\nu\left(\frac{4\pi}{3} - x\right) \Rightarrow$$

$$\frac{\pi}{2} - 2x = 2\kappa\pi + \frac{4\pi}{3} - x$$

$$\Rightarrow -x = 2\kappa\pi + \frac{4\pi}{3} - \frac{\pi}{2}$$

$$\Rightarrow -x = 2\kappa\pi + \frac{5\pi}{6}$$

$$\Rightarrow x = -\frac{5\pi}{6} - 2\kappa\pi$$

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$$\frac{\pi}{2} - 2x = 2\kappa\pi - \frac{4\pi}{3} + x$$

$$\Rightarrow -3x = 2\kappa\pi - \frac{4\pi}{3} - \frac{\pi}{2}$$

$$\Rightarrow -3x = 2\kappa\pi - \frac{11\pi}{6}$$

$$\Rightarrow x = \frac{11\pi}{18} - \frac{2\kappa\pi}{3}$$

Όμως πρέπει οι λύσεις να βρίσκονται στο διάστημα $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Οπότε

- Για $x = \frac{5\pi}{18} - \frac{2\kappa\pi}{3}$ έχουμε

$$\frac{\pi}{2} < \frac{5\pi}{18} - \frac{2\kappa\pi}{3} < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} - \frac{5\pi}{18} < -\frac{2\kappa\pi}{3} < \frac{3\pi}{2} - \frac{5\pi}{18} \Rightarrow \frac{4\pi}{18} < -\frac{2\kappa\pi}{3} < \frac{22\pi}{18} \Rightarrow \frac{1}{3} < -\kappa < \frac{11}{6} \Rightarrow$$

$$\Rightarrow -\frac{11}{6} < \kappa < -\frac{1}{3} \Rightarrow \kappa = -1$$

$$\text{Για } \kappa = -1 \text{ είναι } x = \frac{5\pi}{18} + \frac{2\pi}{3} = \frac{17\pi}{18}$$

- Για $x = \frac{\pi}{6} - 2\kappa\pi$ έχουμε

$$\frac{\pi}{2} < \frac{\pi}{6} - 2\kappa\pi < \frac{3\pi}{2} \Rightarrow \frac{\pi}{3} < -2\kappa\pi < \frac{4\pi}{3} \Rightarrow \frac{1}{6} < -\kappa < \frac{2}{3} \Rightarrow -\frac{2}{3} < \kappa < -\frac{1}{6} \Rightarrow \kappa \notin \mathbb{Z}$$

- Για $x = -\frac{5\pi}{6} - 2\kappa\pi$ έχουμε

$$\frac{\pi}{2} < -\frac{5\pi}{6} - 2\kappa\pi < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} + \frac{5\pi}{6} < -2\kappa\pi < \frac{3\pi}{2} + \frac{5\pi}{6} \Rightarrow \frac{4\pi}{3} < -2\kappa\pi < \frac{7\pi}{3} \Rightarrow \frac{2\pi}{3} < -\kappa\pi < \frac{7\pi}{6} \Rightarrow$$

$$\Rightarrow -\frac{7}{6} < -\kappa < -\frac{2}{3} \Rightarrow \kappa = -1$$

$$\text{Για } \kappa = -1 \text{ είναι } x = -\frac{5\pi}{6} + 2\pi = \frac{7\pi}{6}$$

- Για $x = \frac{11\pi}{18} - \frac{2\kappa\pi}{3}$ έχουμε

$$\frac{\pi}{2} < \frac{11\pi}{18} - \frac{2\kappa\pi}{3} < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} - \frac{11\pi}{18} < -\frac{2\kappa\pi}{3} < \frac{3\pi}{2} - \frac{11\pi}{18} \Rightarrow \frac{-2\pi}{18} < -\frac{2\kappa\pi}{3} < \frac{16\pi}{18} \Rightarrow -\frac{\pi}{6} < -\kappa\pi < \frac{4\pi}{3} \Rightarrow$$

$$\Rightarrow -\frac{4}{3} < \kappa < \frac{1}{6} \Rightarrow \kappa = 0 \text{ ή } \kappa = -1$$

$$\text{για } \kappa = 0 \text{ είναι } x = \frac{11\pi}{18} - 0 = \frac{11\pi}{18}, \text{ για } \kappa = -1 \text{ είναι } x = \frac{11\pi}{18} + \frac{2\pi}{3} = \frac{23\pi}{8}$$

