

Β ΛΥΚΕΙΟΥ ΑΛΓΕΒΡΑ

16.9 1)

a) $2\sigma v^2 x - 1 = 0 \Rightarrow 2\sigma v^2 x = 1 \Rightarrow \sigma v^2 x = \frac{1}{2} \Rightarrow \sigma v x = \pm \sqrt{\frac{1}{2}} \Rightarrow \sigma v x = \pm \frac{1}{\sqrt{2}} \Rightarrow \sigma v x = \pm \frac{\sqrt{2}}{2} \Rightarrow$

$$\Rightarrow \sigma v x = \frac{\sqrt{2}}{2} \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \sigma v x = -\frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow \sigma v x = \sigma v \frac{\pi}{4} \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \sigma v x = -\sigma v \frac{\pi}{4} \Rightarrow \sigma v x = \sigma v \left(\pi - \frac{\pi}{4} \right) \Rightarrow$$

$$\Rightarrow x = 2\kappa\pi + \frac{\pi}{4} \quad \text{ή} \quad x = 2\kappa\pi - \frac{\pi}{4} \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \sigma v x = \sigma v \frac{3\pi}{4} \Rightarrow$$

$$\Rightarrow x = 2\kappa\pi + \frac{3\pi}{4} \quad \text{ή} \quad x = 2\kappa\pi - \frac{3\pi}{4}$$

b) $(4\eta\mu^2 x - 3)(3\epsilon\varphi^2 x - 1) = 0 \Rightarrow (2\eta\mu x - \sqrt{3})(2\eta\mu x + \sqrt{3})(\sqrt{3}\epsilon\varphi x - 1)(\sqrt{3}\epsilon\varphi x + 1) = 0 \Rightarrow$

$$\Rightarrow 2\eta\mu x - \sqrt{3} = 0 \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad 2\eta\mu x + \sqrt{3} = 0 \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \sqrt{3}\epsilon\varphi x - 1 = 0 \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \sqrt{3}\epsilon\varphi x + 1 = 0 \Rightarrow$$

$$\Rightarrow \eta\mu x = \frac{\sqrt{3}}{2} \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \eta\mu x = -\frac{\sqrt{3}}{2} \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \epsilon\varphi x = \frac{1}{\sqrt{3}} \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \epsilon\varphi x = -\frac{1}{\sqrt{3}} \Rightarrow$$

$$\Rightarrow \eta\mu x = \eta\mu \frac{\pi}{3} \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \eta\mu x = \eta\mu \left(-\frac{\pi}{3} \right) \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \epsilon\varphi x = \frac{\sqrt{3}}{3} \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \epsilon\varphi x = -\frac{\sqrt{3}}{3} \Rightarrow$$

$$\Rightarrow x = 2\kappa\pi + \frac{\pi}{3} \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow x = 2\kappa\pi - \frac{\pi}{3} \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \epsilon\varphi x = \epsilon\varphi \frac{\pi}{6} \Rightarrow \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow \epsilon\varphi x = \epsilon\varphi \left(-\frac{\pi}{6} \right) \Rightarrow$$

$$\quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad x = 2\kappa\pi + \frac{2\pi}{3}, \kappa \in \mathbb{Z} \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad x = 2\kappa\pi + \frac{4\pi}{3}, \kappa \in \mathbb{Z} \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow x = \kappa\pi + \frac{\pi}{6}, \kappa \in \mathbb{Z} \quad \begin{array}{l} \text{ή} \\ \parallel \end{array} \quad \Rightarrow x = \kappa\pi - \frac{\pi}{6}, \kappa \in \mathbb{Z}$$

16.9 2)

$\sigma v^2 x - 1 = 0 \Rightarrow \sigma v^2 x = 1 \Rightarrow \sigma v x = \pm 1 \Rightarrow$

$$\begin{aligned} \sigma v x &= 1 \\ \Rightarrow \sigma v x &= \sigma v 0 \\ \Rightarrow x &= 2\kappa\pi \\ \kappa &\in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} \sigma v x &= -1 \\ \Rightarrow \sigma v x &= \sigma v \pi \\ \Rightarrow x &= 2\kappa\pi \pm \pi \\ \kappa &\in \mathbb{Z} \end{aligned}$$

16.9 3)

$\epsilon\varphi^2 x - 3 = 0 \Rightarrow \epsilon\varphi^2 x = 3 \Rightarrow \epsilon\varphi x = \pm\sqrt{3} \Rightarrow$

$$\begin{aligned} \epsilon\varphi x &= \sqrt{3} \\ \Rightarrow \epsilon\varphi x &= \epsilon\varphi \frac{\pi}{6} \\ \Rightarrow x &= \kappa\pi + \frac{\pi}{6} \\ \kappa &\in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} \epsilon\varphi x &= -\sqrt{3} \\ \Rightarrow \epsilon\varphi x &= -\epsilon\varphi \frac{\pi}{6} \Rightarrow \epsilon\varphi x = \epsilon\varphi \left(-\frac{\pi}{6} \right) \\ \Rightarrow x &= \kappa\pi - \frac{\pi}{6} \\ \kappa &\in \mathbb{Z} \end{aligned}$$

16.9 4)

$$2\eta\mu^2x - 1 = 0 \Rightarrow 2\eta\mu^2x = 1 \Rightarrow \eta\mu^2x = \frac{1}{2} \Rightarrow \eta\mu x = \pm \frac{\sqrt{2}}{2} \Rightarrow$$

$$\eta\mu x = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \eta\mu x = \eta\mu \frac{\pi}{4}$$

$$\Rightarrow x = 2\kappa\pi + \frac{\pi}{4} \quad \text{ñ} \quad x = 2\kappa\pi + \frac{3\pi}{4}$$

$\kappa \in \mathbb{Z}$

ñ

$$\eta\mu x = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow \eta\mu x = -\eta\mu \frac{\pi}{4} \Rightarrow \eta\mu x = \eta\mu \left(-\frac{\pi}{4}\right)$$

$$\Rightarrow x = 2\kappa\pi - \frac{\pi}{4} \quad \text{ñ} \quad \eta\mu x = 2\kappa\pi + \frac{5\pi}{4}$$

$\kappa \in \mathbb{Z}$

16.9 5)

$$3\sigma\varphi^2x - 1 = 0 \Rightarrow 3\sigma\varphi^2x = 1 \Rightarrow \sigma\varphi^2x = \frac{1}{3} \Rightarrow \sigma\varphi x = \pm \frac{\sqrt{3}}{3} \Rightarrow$$

$$\sigma\varphi x = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{3}$$

$$\Rightarrow x = \kappa\pi + \frac{\pi}{3}$$

$\kappa \in \mathbb{Z}$

ñ

$$\sigma\varphi x = -\frac{\sqrt{3}}{3}$$

$$\Rightarrow \sigma\varphi x = -\sigma\varphi \frac{\pi}{3} \Rightarrow \sigma\varphi x = \sigma\varphi \left(-\frac{\pi}{3}\right)$$

$$\Rightarrow x = \kappa\pi - \frac{\pi}{3}$$

$\kappa \in \mathbb{Z}$

16.9 6)

$$(2\sigma v v^2 x - 1)(\sigma\varphi^2 x - 3) = 0 \Rightarrow (\sqrt{2}\sigma v v - 1)(\sqrt{2}\sigma v v x + 1)(\sigma\varphi x - \sqrt{3})(\sigma\varphi x + \sqrt{3}) = 0$$

$$\sqrt{2}\sigma v v x - 1 = 0$$

$$\Rightarrow \sqrt{2}\sigma v v x = 1$$

$$\Rightarrow \sigma v v x = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \sigma v v x = \sigma v v \frac{\pi}{4}$$

$$\Rightarrow x = 2\kappa\pi \pm \frac{\pi}{4}$$

$\kappa \in \mathbb{Z}$

$$\sqrt{2}\sigma v v x + 1 = 0$$

$$\Rightarrow \sqrt{2}\sigma v v x = -1$$

$$\Rightarrow \sigma v v x = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow \sigma v v x = \sigma v v \left(\pi - \frac{\pi}{4}\right)$$

$$\Rightarrow x = 2\kappa\pi \pm \frac{3\pi}{4}$$

$\kappa \in \mathbb{Z}$

$$\sigma\varphi x - \sqrt{3} = 0$$

$$\Rightarrow \sigma\varphi x = \sqrt{3}$$

$$\Rightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{6}$$

$$\Rightarrow x = \kappa\pi + \frac{\pi}{6}$$

$\kappa \in \mathbb{Z}$

$$\sigma\varphi x + \sqrt{3} = 0$$

$$\Rightarrow \sigma\varphi x = -\sqrt{3}$$

$$\Rightarrow \sigma\varphi x = -\sigma\varphi \frac{\pi}{6}$$

$$\Rightarrow \sigma\varphi x = \sigma\varphi \left(-\frac{\pi}{6}\right)$$

$$\Rightarrow x = \kappa\pi - \frac{\pi}{6}$$

$\kappa \in \mathbb{Z}$

16.9 7)

$$(\eta\mu^2x - 1)(2\sigma\upsilon\nu^2x - 1) = 0 \Rightarrow (\eta\mu x - 1)(\eta\mu x + 1)(2\sigma\upsilon\nu x - 1)(2\sigma\upsilon\nu x + 1) = 0$$

$$\begin{aligned} \eta\mu x - 1 &= 0 \\ \Rightarrow \eta\mu x &= 1 \\ \Rightarrow \eta\mu x &= \eta\mu \frac{\pi}{2} \\ \Rightarrow x &= 2\kappa\pi + \frac{\pi}{2} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

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$$\begin{aligned} \eta\mu x + 1 &= 0 \\ \Rightarrow \eta\mu x &= -1 \\ \Rightarrow \eta\mu x &= -\eta\mu \frac{\pi}{2} \\ \Rightarrow \eta\mu x &= \eta\mu \left(-\frac{\pi}{2}\right) \\ \Rightarrow x &= 2\kappa\pi + \frac{\pi}{2} \\ &\quad \checkmark \\ x &= 2\kappa\pi + \frac{3\pi}{2} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} 2\sigma\upsilon\nu x - 1 &= 0 \\ \Rightarrow \sigma\upsilon\nu x &= \frac{1}{2} \\ \Rightarrow \sigma\upsilon\nu x &= \sigma\upsilon\nu \frac{\pi}{3} \\ \Rightarrow x &= 2\kappa\pi \pm \frac{\pi}{3} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

η

$$\begin{aligned} 2\sigma\upsilon\nu x + 1 &= 0 \\ \Rightarrow \sigma\upsilon\nu x &= -\frac{1}{2} \\ \Rightarrow \sigma\upsilon\nu x &= -\sigma\upsilon\nu \frac{\pi}{3} \\ \Rightarrow x &= 2\kappa\pi \pm \frac{2\pi}{3} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

η

16.9 8)

$$(4\sigma\upsilon\nu^2x - 3)(\epsilon\varphi^2x - 1) = 0 \Rightarrow (2\sigma\upsilon\nu x - \sqrt{3})(2\sigma\upsilon\nu x + \sqrt{3})(\epsilon\varphi x - 1)(\epsilon\varphi x + 1) = 0$$

$$\begin{aligned} 2\sigma\upsilon\nu x - \sqrt{3} &= 0 \\ \Rightarrow \sigma\upsilon\nu x &= \frac{\sqrt{3}}{2} \\ \Rightarrow \sigma\upsilon\nu x &= \sigma\upsilon\nu \frac{\pi}{6} \\ \Rightarrow x &= 2\kappa\pi \pm \frac{\pi}{6} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

η

$$\begin{aligned} 2\sigma\upsilon\nu x + \sqrt{3} &= 0 \\ \Rightarrow \sigma\upsilon\nu x &= -\frac{\sqrt{3}}{2} \\ \Rightarrow \sigma\upsilon\nu x &= -\sigma\upsilon\nu \frac{\pi}{6} \\ \Rightarrow \sigma\upsilon\nu x &= \sigma\upsilon\nu \frac{5\pi}{6} \\ \Rightarrow x &= 2\kappa\pi \pm \frac{5\pi}{6} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

η

$$\begin{aligned} \epsilon\varphi x - 1 &= 0 \\ \Rightarrow \epsilon\varphi x &= 1 \\ \Rightarrow \epsilon\varphi x &= \epsilon\varphi \frac{\pi}{4} \\ \Rightarrow x &= \kappa\pi + \frac{\pi}{4} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

η

$$\begin{aligned} \epsilon\varphi x + 1 &= 0 \\ \Rightarrow \epsilon\varphi x &= -1 \\ \Rightarrow \epsilon\varphi x &= -\epsilon\varphi \frac{\pi}{4} \\ \Rightarrow \epsilon\varphi x &= \epsilon\varphi \left(-\frac{\pi}{4}\right) \\ \Rightarrow x &= \kappa\pi - \frac{\pi}{4} \end{aligned}$$

16.9 9)

$$(4\eta\mu^2x - 1)(\sigma\varphi^2x - 1) = 0 \Rightarrow (2\eta\mu x - 1)(2\eta\mu x + 1)(\sigma\varphi x - 1)(\sigma\varphi x + 1) = 0$$

$$\begin{aligned} 2\eta\mu x - 1 &= 0 \\ \Rightarrow \eta\mu x &= \frac{1}{2} \\ \Rightarrow \eta\mu x &= \eta\mu \frac{\pi}{6} \\ \Rightarrow x &= 2\kappa\pi + \frac{\pi}{6} \\ &\quad \checkmark \\ x &= 2\kappa\pi + \frac{5\pi}{6} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

η

$$\begin{aligned} 2\eta\mu x + 1 &= 0 \\ \Rightarrow \eta\mu x &= -\frac{1}{2} \\ \Rightarrow \eta\mu x &= \eta\mu \left(-\frac{\pi}{6}\right) \\ \Rightarrow x &= 2\kappa\pi - \frac{\pi}{6} \\ &\quad \checkmark \\ x &= 2\kappa\pi + \frac{7\pi}{6} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

η

$$\begin{aligned} \sigma\varphi x - 1 &= 0 \\ \Rightarrow \sigma\varphi x &= 1 \\ \Rightarrow \sigma\varphi x &= \sigma\varphi \frac{\pi}{4} \\ \Rightarrow x &= \kappa\pi + \frac{\pi}{4} \\ &\quad \kappa \in \mathbb{Z} \end{aligned}$$

η

$$\begin{aligned} \sigma\varphi x + 1 &= 0 \\ \Rightarrow \sigma\varphi x &= -1 \\ \Rightarrow \sigma\varphi x &= \sigma\varphi \left(-\frac{\pi}{4}\right) \\ \Rightarrow x &= \kappa\pi - \frac{\pi}{4} \end{aligned}$$