

16.18 1)

$$\begin{aligned}\varepsilon\varphi\left(x + \frac{\pi}{6}\right) &= \sigma\varphi x \Rightarrow \varepsilon\varphi\left(x + \frac{\pi}{6}\right) = \varepsilon\varphi\left(\frac{\pi}{2} - x\right) \Rightarrow x + \frac{\pi}{6} = \kappa\pi + \frac{\pi}{2} - x \Rightarrow 2x = \kappa\pi + \frac{\pi}{2} - \frac{\pi}{6} \Rightarrow \\ \Rightarrow 2x &= \kappa\pi + \frac{2\pi}{6} \Rightarrow 2x = \kappa\pi + \frac{\pi}{3} \Rightarrow x = \frac{\kappa\pi}{2} + \frac{\pi}{6}\end{aligned}$$

$$\begin{aligned}\text{Όμως πρέπει } x \in (\pi, 2\pi) &\Rightarrow \pi < x < 2\pi \Rightarrow \pi < \frac{\kappa\pi}{2} + \frac{\pi}{6} < 2\pi \Rightarrow 6\pi < 3\kappa\pi + \pi < 12\pi \Rightarrow 6 < 3\kappa + 1 < 12 \Rightarrow \\ \Rightarrow 5 < 3\kappa < 11 &\Rightarrow \frac{5}{3} < \kappa < \frac{11}{3}. \text{ Και επειδή ο } \kappa \text{ είναι ακέραιος θα είναι } \kappa = 2 \text{ ή } \kappa = 3\end{aligned}$$

$$\text{Για } \kappa = 2 \text{ έχουμε } x = \frac{2\pi}{2} + \frac{\pi}{6} \Rightarrow x = \frac{5\pi}{6} \text{ και για } \kappa = 3 \text{ έχουμε } x = \frac{3\pi}{2} + \frac{\pi}{6} \Rightarrow x = \frac{10\pi}{6} = \frac{5\pi}{3}$$

16.18 2)

$$\eta\mu\left(x - \frac{\pi}{3}\right) = \eta\mu\left(2x + \frac{\pi}{6}\right) \Rightarrow$$

$$\Rightarrow x - \frac{\pi}{3} = 2\kappa\pi + 2x + \frac{\pi}{6}$$

$$\Rightarrow x = -2\kappa\pi - \frac{\pi}{2}$$

$$\text{Όμως πρέπει } 0 < x < 2\pi$$

$$\Rightarrow 0 < 2\kappa\pi - \frac{\pi}{2} < 2\pi$$

$$\Rightarrow \frac{\pi}{2} < -2\kappa\pi < \frac{5\pi}{2}$$

$$\Rightarrow -\frac{5}{4} < \kappa < -\frac{\pi}{4}$$

$$\text{και επειδή ο } \kappa \text{ είναι ακέραιος} \\ \text{θα είναι } \kappa = -1 \text{ οπότε}$$

$$x = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

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$$\Rightarrow x - \frac{\pi}{3} = 2\kappa\pi + \pi - \left(2x + \frac{\pi}{6}\right)$$

$$\Rightarrow x - \frac{\pi}{3} = 2\kappa\pi + \pi - 2x - \frac{\pi}{6}$$

$$\Rightarrow 3x = 2\kappa\pi + \frac{7\pi}{6} \Rightarrow x = \frac{2\kappa\pi}{3} + \frac{7\pi}{18}$$

$$\text{Όμως πρέπει } 0 < x < 2\pi \Rightarrow 0 < \frac{2\kappa\pi}{3} + \frac{7\pi}{18} < 2\pi$$

$$\Rightarrow \frac{-7\pi}{18} < \frac{2\kappa\pi}{3} < \frac{29\pi}{18} \Rightarrow \frac{-7\pi}{12} < \kappa < \frac{29}{12}$$

$$\text{και επειδή ο } \kappa \text{ είναι ακέραιος θα είναι} \\ \kappa = 2 \text{ ή } \kappa = 1 \text{ ή } \kappa = 2$$

$$\text{Για } \kappa = 0, \quad x = \frac{7\pi}{18}$$

$$\text{Για } \kappa = 1, \quad x = \frac{19\pi}{18}$$

$$\text{Για } \kappa = 2, \quad x = \frac{31\pi}{18}$$

16.18 3)

$$\sigma\varphi\left(x - \frac{\pi}{4}\right) = -\sigma\varphi\left(x + \frac{3\pi}{4}\right) \Rightarrow \sigma\varphi\left(x - \frac{\pi}{4}\right) = \sigma\varphi\left(-x - \frac{3\pi}{4}\right) \Rightarrow$$

$$\Rightarrow x - \frac{\pi}{4} = \kappa\pi - x - \frac{3\pi}{4} \Rightarrow 2x = \kappa\pi - \frac{2\pi}{4} \Rightarrow x = \frac{\kappa\pi}{2} - \frac{\pi}{4}$$

$$\text{Όμως πρέπει : } 0 < x < \pi \Rightarrow 0 < \frac{\kappa\pi}{2} - \frac{\pi}{4} < \pi \Rightarrow \frac{\pi}{4} < \frac{\kappa\pi}{2} < \frac{5\pi}{4} \Rightarrow$$

$$\Rightarrow \frac{1}{2} < \kappa < \frac{5}{2}$$

$$\text{Και επειδή ο } \kappa \text{ είναι ακέραιος, θα είναι } \kappa = 1 \text{ ή } \kappa = 2$$

$$\text{Για } \kappa = 1 \text{ είναι } x = \frac{\pi}{4}$$

$$\text{Για } \kappa = 2 \text{ είναι } x = \frac{3\pi}{4}$$

$$\eta\mu\left(x - \frac{\pi}{3}\right) = \eta\mu\left(2x + \frac{\pi}{6}\right) \Rightarrow$$

$$\frac{\pi}{2} - x = 2\kappa\pi + x + \frac{2\pi}{3}$$

$$\Rightarrow 2x = -2\kappa\pi + \frac{\pi}{2} - \frac{2\pi}{3} \Rightarrow 2x = -2\kappa\pi - \frac{\pi}{6}$$

$$\Rightarrow x = -\kappa\pi - \frac{\pi}{12}$$

Όμως πρέπει $x \in [-\pi, \pi]$

$$\Rightarrow -\pi \leq x \leq \pi \Rightarrow -\pi \leq -\kappa\pi - \frac{\pi}{12} \leq \pi$$

$$\Rightarrow -\pi \leq \kappa\pi + \frac{\pi}{12} \leq \pi \Rightarrow -\frac{13\pi}{12} \leq \kappa\pi \leq \frac{11\pi}{12}$$

$$\Rightarrow -\frac{13}{12} \leq \kappa \leq \frac{11}{12}$$

Και επειδή ο κ είναι ακέραιος, θα είναι
 $\kappa = 0$ ή $\kappa = -1$

$$\text{Για } \kappa = 0, \quad x = -\frac{\pi}{12}$$

$$\text{Για } \kappa = -1, \quad x = \frac{11\pi}{12}$$

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$$\Rightarrow \frac{\pi}{2} - x = 2\kappa\pi - x - \frac{2\pi}{3}$$

$\Rightarrow$ αδύνατη