

16.15 1)

$$1 + 2\sigma\upsilon\nu x - \eta\mu x = 2\eta\mu x \sigma\upsilon\nu x \Leftrightarrow 1 + 2\sigma\upsilon\nu x - \eta\mu x - 2\eta\mu x \sigma\upsilon\nu x = 0 \Leftrightarrow$$

$$\Leftrightarrow 1 + 2\sigma\upsilon\nu x - \eta\mu x (1 + 2\sigma\upsilon\nu x) = 0 \Leftrightarrow (1 + 2\sigma\upsilon\nu x)(1 - \eta\mu x) = 0 \Leftrightarrow$$

$\Leftrightarrow 1 + 2\sigma\upsilon\nu x = 0 \Leftrightarrow$ $\Leftrightarrow 2\sigma\upsilon\nu x = -1 \Rightarrow \sigma\upsilon\nu x = -\frac{1}{2} \Leftrightarrow$ $\Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu\left(\pi - \frac{\pi}{3}\right) \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{2\pi}{3} \Leftrightarrow$ $\Leftrightarrow x = 2\kappa\pi + \frac{2\pi}{3} \quad \text{ή} \quad x = 2\kappa\pi - \frac{2\pi}{3}, \quad \kappa \in \mathbb{Z}$		$\text{ή} \quad 1 - \eta\mu x = 0 \Leftrightarrow$ $\Rightarrow \eta\mu x = 1 \Rightarrow \eta\mu x = \eta\mu \frac{\pi}{2} \Rightarrow$ $\Rightarrow x = 2\kappa\pi + \frac{\pi}{2} \quad \text{ή} \quad x = 2\kappa\pi + \pi - \frac{\pi}{2} = 2\kappa\pi + \frac{\pi}{2}, \quad \kappa \in \mathbb{Z}$
--	---	---

16.15 2)

$$4\eta\mu x \cdot \sigma\upsilon\nu x - 2\eta\mu x - 2\sigma\upsilon\nu x + 1 = 0 \Rightarrow 2\eta\mu x (2\sigma\upsilon\nu x - 1) - 2\sigma\upsilon\nu x + 1 = 0 \Rightarrow$$

$$\Rightarrow (2\sigma\upsilon\nu x - 1)(2\eta\mu x - 1) = 0 \Rightarrow$$

$2\sigma\upsilon\nu x - 1 = 0$ $\Rightarrow 2\sigma\upsilon\nu x = 1$ $\Rightarrow \sigma\upsilon\nu x = \frac{1}{2}$ $\Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{3}$ $\Rightarrow x = 2\kappa\pi \pm \frac{\pi}{3}, \quad \kappa \in \mathbb{Z}$		$2\eta\mu x - 1 = 0$ $\Rightarrow 2\eta\mu x = 1$ $\Rightarrow \eta\mu x = \frac{1}{2}$ $\Rightarrow \eta\mu x = \eta\mu \frac{\pi}{6}$ $\Rightarrow x = 2\kappa\pi + \frac{\pi}{6}$ ή $x = 2\kappa\pi + \frac{5\pi}{6}, \quad \kappa \in \mathbb{Z}$
---	--	--

16.15 3)

$$\sigma\varphi x \cdot \sigma\upsilon\nu x + 1 = \sigma\upsilon\nu x + \sigma\varphi x \Rightarrow \sigma\varphi x \cdot \sigma\upsilon\nu x + 1 - \sigma\upsilon\nu x - \sigma\varphi x = 0 \Rightarrow$$

$$\Rightarrow \sigma\varphi x (\sigma\upsilon\nu x - 1) + 1 - \sigma\upsilon\nu x = 0 \Rightarrow (\sigma\upsilon\nu x - 1)(\sigma\varphi x - 1) = 0 \Rightarrow$$

$\Rightarrow \sigma\upsilon\nu x - 1 = 0$ $\Rightarrow \sigma\upsilon\nu x = 1$ $\Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu 0$ $\Rightarrow x = 2\kappa\pi, \quad \kappa \in \mathbb{Z}$ απορρίπτεται γιατί η σφx δεν ορίζεται για $x = \kappa\pi, \quad \kappa \in \mathbb{Z}$		$\Rightarrow \sigma\varphi x - 1 = 0$ $\Rightarrow \sigma\varphi x = 1$ $\Rightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{4}$ $\Rightarrow x = \kappa\pi + \frac{\pi}{4}, \quad \kappa \in \mathbb{Z}$
--	--	--

16.15 4)

$$\eta\mu x + \varepsilon\varphi x = 1 + \eta\mu x \cdot \varepsilon\varphi x \Rightarrow \eta\mu x + \varepsilon\varphi x - 1 - \eta\mu x \cdot \varepsilon\varphi x = 0 \Rightarrow$$

$$\Rightarrow \eta\mu x(1 - \varepsilon\varphi x) + \varepsilon\varphi x - 1 = 0 \Rightarrow (1 - \varepsilon\varphi x)(\eta\mu x - 1) = 0 \Rightarrow$$

$$\varepsilon\varphi x = 1$$

ή

$$\Rightarrow \eta\mu x = 1$$

$$\Rightarrow \varepsilon\varphi x = \varepsilon\varphi \frac{\pi}{4}$$

$$\Rightarrow \eta\mu x = \eta\mu \frac{\pi}{2}$$

$$\Rightarrow x = \kappa\pi + \frac{\pi}{4}, \kappa \in \mathbb{Z}$$

$$\Rightarrow x = 2\kappa\pi + \frac{\pi}{2}, \kappa \in \mathbb{Z}$$

απορρίπτεται γιατί η $\varepsilon\varphi x$
δεν ορίζεται για

$$x = 2\kappa\pi + \frac{\pi}{2}, \kappa \in \mathbb{Z}$$

16.15 5)

$$\sin^3 x - 2\sin^2 x - \sin x + 2 = 0 \Rightarrow \sin^3 x - \sin x - 2\sin^2 x + 2 = 0 \Rightarrow$$

$$\Rightarrow \sin x(\sin^2 x - 1) - 2(\sin^2 x - 1) = 0 \Rightarrow (\sin^2 x - 1)(\sin x - 2) = 0 \Rightarrow$$

$$\Rightarrow (\sin x - 1)(\sin x + 1)(\sin x - 2) = 0 \Rightarrow$$

$$\Rightarrow \sin^2 x = 1$$

ή

$$\sin x = -1$$

ή

$$\Rightarrow \sin x = 2$$

$$\Rightarrow \sin x = 1$$

$$\Rightarrow \sin x = \sin \pi$$

$$\Rightarrow \text{αδύνατη}$$

$$\Rightarrow \sin x = \sin 0$$

$$\Rightarrow x = 2\kappa\pi \pm \pi$$

$$\Rightarrow x = 2\kappa\pi$$

Συνοψίζοντας τις παραπάνω λύσεις έχουμε $x = \kappa\pi$, $\kappa \in \mathbb{Z}$