16.14 1)

- Για να λύσουμε αυτή την άσκηση θα χρειαστεί να θυμηθούμε από την παράγραφο **«αναγωγή στο πρώτο τεταρτημόριο» τους τύπους**

$$\sigma\upsilon\nu\omega = \eta\mu\left(\frac{\pi}{2}-\omega\right) \qquad \eta\mu\omega = \sigma\upsilon\nu\left(\frac{\pi}{2}-\omega\right) \qquad \sigma\varphi\omega = \varepsilon\varphi\left(\frac{\pi}{2}-\omega\right) \qquad \varepsilon\varphi\omega = \sigma\varphi\left(\frac{\pi}{2}-\omega\right)$$

α) $\eta\mu 2x = \sigma\upsilon\nu x \Rightarrow \eta\mu 2x = \eta\mu\left(\frac{\pi}{2}-x\right) \Rightarrow$

$\Rightarrow 2x = 2\kappa\pi + \frac{\pi}{2} - x \Rightarrow$

$\Rightarrow 2x + x = 2\kappa\pi + \frac{\pi}{2} \Rightarrow 3x = 2\kappa\pi + \frac{\pi}{2} \Rightarrow$

$\Rightarrow x = \frac{2\kappa\pi}{3} + \frac{\pi}{6}\ ,\ \kappa \in Z$

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$2x = 2\kappa\pi + \pi - \left(\frac{\pi}{2} - x\right)$

$\Rightarrow 2x = 2\kappa\pi + \pi - \frac{\pi}{2} + x \Rightarrow 2x - x = 2\kappa\pi + \pi - \frac{\pi}{2} \Rightarrow$

$\Rightarrow x = 2\kappa\pi + \frac{\pi}{2}\ ,\ \kappa \in Z$

β) $\sigma\varphi\left(2x+\frac{\pi}{3}\right) = -\varepsilon\varphi\left(x+\frac{\pi}{6}\right) \Rightarrow \varepsilon\varphi\left[\frac{\pi}{2}-\left(2x+\frac{\pi}{3}\right)\right] = \varepsilon\varphi\left[-\left(x+\frac{\pi}{6}\right)\right] \Rightarrow$

$\Rightarrow \varepsilon\varphi\left(\frac{\pi}{2}-2x-\frac{\pi}{3}\right) = \varepsilon\varphi\left(-x-\frac{\pi}{6}\right) \Rightarrow \varepsilon\varphi\left(\frac{\pi}{6}-2x\right) = \varepsilon\varphi\left(-x-\frac{\pi}{6}\right) \Rightarrow$

$\Rightarrow \frac{\pi}{6} - 2x = \kappa\pi - x - \frac{\pi}{6} \Rightarrow -2x + x = \kappa\pi - \frac{\pi}{6} - \frac{\pi}{6} \Rightarrow -x = \kappa\pi - \frac{2\pi}{3} \Rightarrow x = -\kappa\pi + \frac{\pi}{3}$

16.14 2)

$\eta\mu x = \sigma\upsilon\nu x \Rightarrow \eta\mu x = \eta\mu\left(\frac{\pi}{2}-x\right) \Rightarrow$

$x = 2\kappa\pi + \frac{\pi}{2} - x$

$\Rightarrow 2x = 2\kappa\pi + \frac{\pi}{2}$

$\Rightarrow x = \kappa\pi + \frac{\pi}{4}\ ,\ \kappa \in Z$

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$x = 2\kappa\pi + \pi - \frac{\pi}{2}$

$\Rightarrow$ αδύνατη

16.14 3)

$\varepsilon\varphi x = \sigma\varphi x \Rightarrow \varepsilon\varphi x = \varepsilon\varphi\left(\frac{\pi}{2}-x\right) \Rightarrow x = \kappa\pi + \frac{\pi}{2} - x \Rightarrow 2x = \kappa\pi + \frac{\pi}{2} \Rightarrow x = \frac{\kappa\pi}{2} + \frac{\pi}{2}\ ,\ \kappa \in Z$

16.14 4)

$\eta\mu 2x = \sigma\upsilon\nu 3x \Rightarrow \eta\mu 2x = \eta\mu\left(\frac{\pi}{2}-3x\right) \Rightarrow$

$\Rightarrow 2x = 2\kappa\pi + \frac{\pi}{2} - 3x$

$\Rightarrow 5x = 2\kappa\pi + \frac{\pi}{2}$

$\Rightarrow x = \frac{2\kappa\pi}{5} + \frac{\pi}{10}\ ,\ \kappa \in Z$

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$\Rightarrow 2x = 2\kappa\pi + \pi - \frac{\pi}{2} + 3x$

$\Rightarrow -x = 2\kappa\pi + \frac{\pi}{2}$

$\Rightarrow x = -2\kappa\pi - \frac{\pi}{2}\ ,\ \kappa \in Z$

16.14 5)

$$\sigma\upsilon\nu\left(2x+\frac{\pi}{4}\right)=-\eta\mu x \Rightarrow \sigma\upsilon\nu\left(2x+\frac{\pi}{4}\right)=\eta\mu(-x) \Rightarrow \sigma\upsilon\nu\left(2x+\frac{\pi}{4}\right)=\sigma\upsilon\nu\left(\frac{\pi}{2}-(-x)\right) \Rightarrow$$

$$\Rightarrow \sigma\upsilon\nu\left(2x+\frac{\pi}{4}\right)=\sigma\upsilon\nu\left(\frac{\pi}{2}+x\right) \Rightarrow 2x+\frac{\pi}{4}=2\kappa\pi\pm\left(\frac{\pi}{2}+x\right) \Rightarrow$$

$$\Rightarrow 2x+\frac{\pi}{4}=2\kappa\pi+\frac{\pi}{2}+x$$

$$\Rightarrow x=2\kappa\pi+\frac{\pi}{4}\ ,\ \kappa\in Z$$

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$$\Rightarrow 2x+\frac{\pi}{4}=2\kappa\pi-\frac{\pi}{2}-x$$

$$\Rightarrow 3x=2\kappa\pi-\frac{3\pi}{4}$$

$$\Rightarrow x=\frac{2\kappa\pi}{3}-\frac{\pi}{4}\ ,\ \kappa\in Z$$

16.14 6)

$$\eta\mu 2x \Rightarrow \sigma\upsilon\nu\left(\frac{\pi}{3}-x\right) \Rightarrow \eta\mu 2x=\eta\mu\left[\frac{\pi}{2}-\left(\frac{\pi}{3}-x\right)\right] \Rightarrow \eta\mu 2x=\eta\mu\left(\frac{\pi}{6}+x\right) \Rightarrow$$

$$2x=2\kappa\pi+\frac{\pi}{6}+x$$

$$\Rightarrow x=2\kappa\pi+\frac{\pi}{6}\ ,\ \kappa\in Z$$

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$$2x=2\kappa\pi+\pi-\frac{\pi}{6}-x$$

$$\Rightarrow 3x=2\kappa\pi+\frac{5\pi}{6}$$

$$\Rightarrow x=\frac{2\kappa\pi}{3}+\frac{5\pi}{18}\ ,\ \kappa\in Z$$

16.14 7)

$$\varepsilon\varphi x=\sigma\varphi\left(3x+\frac{\pi}{6}\right)=0 \Rightarrow \varepsilon\varphi x=-\sigma\varphi\left(3x+\frac{\pi}{6}\right) \Rightarrow \varepsilon\varphi x=\sigma\varphi\left(-3x-\frac{\pi}{6}\right) \Rightarrow$$

$$\Rightarrow \varepsilon\varphi x=\varepsilon\varphi\left[\frac{\pi}{2}-\left(-3x-\frac{\pi}{6}\right)\right] \Rightarrow \varepsilon\varphi x=\varepsilon\varphi\left(\frac{\pi}{2}+3x+\frac{\pi}{6}\right) \Rightarrow \varepsilon\varphi x=\varepsilon\varphi\left(\frac{2\pi}{3}+3x\right) \Rightarrow$$

$$x=\kappa\pi+\frac{2\pi}{3}+3x \Rightarrow -2x=\kappa\pi+\frac{2\pi}{3} \Rightarrow x=-\kappa\pi-\frac{\pi}{3}\ ,\ \kappa\in Z$$

16.14 8)

$$\varepsilon\varphi\left(2x+\frac{\pi}{3}\right)-\sigma\varphi\left(3x-\frac{\pi}{4}\right)=0 \Rightarrow \varepsilon\varphi\left(2x+\frac{\pi}{3}\right)=\sigma\varphi\left(3x-\frac{\pi}{4}\right) \Rightarrow$$

$$\Rightarrow \varepsilon\varphi\left(2x+\frac{\pi}{3}\right)=\varepsilon\varphi\left(\frac{\pi}{2}-3x+\frac{\pi}{4}\right) \Rightarrow \varepsilon\varphi\left(2x+\frac{\pi}{3}\right)=\varepsilon\varphi\left(\frac{3\pi}{4}-3x\right) \Rightarrow$$

$$\Rightarrow 2x+\frac{\pi}{3}=\kappa\pi+\frac{3\pi}{4}-3x \Rightarrow 5x=\kappa\pi+\frac{3\pi}{4}-\frac{\pi}{3} \Rightarrow$$

$$\Rightarrow 5x=\kappa\pi+\frac{5\pi}{12} \Rightarrow x=\frac{\kappa\pi}{5}+\frac{\pi}{12}\ ,\ \kappa\in Z$$