

16.13 1)

$$\Delta = 25 - 16 = 9, \quad y_{1,2} = \frac{5 \pm \sqrt{9}}{2 \cdot 2} = \frac{5 \pm 3}{4} \Rightarrow \begin{cases} y_1 = \frac{5-3}{4} = \frac{2}{4} = \frac{1}{2} \\ y_2 = \frac{5+3}{4} = 2 \end{cases}$$

α) Θέτουμε $\eta\mu x = y$ και η εξίσωση γίνεται $2y^2 - 5y + 2 = 0 \Rightarrow$

$$\Rightarrow y = \frac{1}{2}$$

$$\Rightarrow \eta\mu x = \frac{1}{2} \Rightarrow \eta\mu x = \eta\mu \frac{\pi}{6} \Rightarrow$$

$$\Rightarrow x = 2\kappa\pi + \frac{\pi}{6} \quad \text{ή} \quad x = 2\kappa\pi + \pi - \frac{\pi}{6} = 2\kappa\pi + \frac{5\pi}{6}, \quad \kappa \in \mathbb{Z}$$

$$\text{ή} \Rightarrow y = 2$$

$$\Rightarrow \eta\mu x = 2, \text{ αδύνατη}$$

β) Θέτουμε $\epsilon\phi x = y$ και η εξίσωση γίνεται

$$\Delta = 12 + 36 = 48, \quad y_{1,2} = \frac{-2\sqrt{3} \pm \sqrt{48}}{2 \cdot 3} = \frac{-2\sqrt{3} \pm 4\sqrt{3}}{6} \Rightarrow \begin{cases} y_1 = \frac{-2\sqrt{3} + 4\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3} \\ y_2 = \frac{-2\sqrt{3} - 4\sqrt{3}}{6} = \frac{-6\sqrt{3}}{6} = -\sqrt{3} \end{cases}$$

$$3y^2 + 2\sqrt{3}y - 3 = 0 \Rightarrow$$

$$\Rightarrow y = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \epsilon\phi x = \frac{\sqrt{3}}{3} \Rightarrow \epsilon\phi x = \epsilon\phi \frac{\pi}{6} \Rightarrow x = \kappa\pi + \frac{\pi}{6}, \quad \kappa \in \mathbb{Z}$$

$$\text{ή} \quad y = -\sqrt{3} \Rightarrow$$

$$\Rightarrow \epsilon\phi x = -\sqrt{3} \Rightarrow \epsilon\phi x = \epsilon\phi \left(-\frac{\pi}{3} \right) \Rightarrow$$

$$\Rightarrow x = \kappa\pi - \frac{\pi}{3}, \quad \kappa \in \mathbb{Z}$$

16.13 2)

Θέτουμε $\eta\mu x = y$ και η εξίσωση γράφεται : $2y^2 + 7y - 4 = 0 \Rightarrow y_{1,2} = \frac{-7 \pm \sqrt{81}}{4} \Rightarrow \begin{cases} -4 \\ \frac{1}{2} \end{cases}$

$$\Rightarrow y = -4$$

$$\Rightarrow \eta\mu x = -4$$

$$\Rightarrow \text{αδύνατη}$$

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$$\Rightarrow y = \frac{1}{2}$$

$$\Rightarrow \eta\mu x = \frac{1}{2}$$

$$\Rightarrow x = 2\kappa\pi + \frac{\pi}{6}$$

$$\text{ή} \quad x = 2\kappa\pi + \pi - \frac{\pi}{6}, \quad \kappa \in \mathbb{Z}$$

16.13 3)

Θέτουμε $y = \sigma\upsilon\nu x$ και η εξίσωση γράφεται : $2y^2 - 3y + 1 = 0 \Rightarrow y_{1,2} = \frac{3 \pm 1}{4} \Rightarrow \begin{cases} \frac{1}{2} \\ 1 \end{cases}$

$$\Rightarrow y = 1$$

$$\Rightarrow \sigma\upsilon\nu x = 1$$

$$\Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu 0$$

$$\Rightarrow x = 2\kappa\pi, \quad \kappa \in \mathbb{Z}$$

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$$\Rightarrow y = \frac{1}{2}$$

$$\Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{3}$$

$$\Rightarrow x = 2\kappa\pi \pm \frac{\pi}{3}, \quad \kappa \in \mathbb{Z}$$

16.13 4)

Θέτουμε $y = \varepsilon\varphi x$ και η εξίσωση γράφεται : $y^2 - y = 0 \Rightarrow y(y-1) = 0 \Rightarrow$

$$y = 0$$

$$\Rightarrow \varepsilon\varphi x = 0$$

$$\Rightarrow \varepsilon\varphi x = \varepsilon\varphi 0$$

$$\Rightarrow x = \kappa\pi, \kappa \in \mathbb{Z}$$

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$$y = 1$$

$$\Rightarrow \varepsilon\varphi x = 1$$

$$\Rightarrow \varepsilon\varphi x = \varepsilon\varphi \frac{\pi}{4}$$

$$\Rightarrow x = \kappa\pi + \frac{\pi}{4}, \kappa \in \mathbb{Z}$$

16.13 5)

Θέτουμε $y = \sigma\varphi x$ και η εξίσωση γράφεται : $3y^2 - 4\sqrt{3}y + 3 = 0$

$$\stackrel{\Delta=12}{\Rightarrow} y_{1,2} = \frac{4\sqrt{3} \pm \sqrt{12}}{6} = \frac{4\sqrt{3} \pm 2\sqrt{3}}{6} \begin{matrix} \nearrow y_1 = \sqrt{3} \\ \searrow y_2 = \frac{\sqrt{3}}{3} \end{matrix}$$

$$y = \sqrt{3}$$

$$\Rightarrow \sigma\varphi x = \sqrt{3}$$

$$\Rightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{6}$$

$$\Rightarrow x = \kappa\pi + \frac{\pi}{6}, \kappa \in \mathbb{Z}$$

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$$y = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{3}$$

$$\Rightarrow x = \kappa\pi + \frac{\pi}{3}, \kappa \in \mathbb{Z}$$

16.13 6)

Θέτουμε $y = \eta\mu x$ και η εξίσωση γράφεται : $y^2 - y - 6 = 0 \Rightarrow y_{1,2} = \frac{1 \pm \sqrt{25}}{2} \begin{matrix} \nearrow y_1 = 3 \\ \searrow y_2 = -2 \end{matrix}$

$$y = 3$$

$$\Rightarrow \eta\mu x = 3$$

$$\Rightarrow \text{αδύνατη}$$

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$$y = -2$$

$$\Rightarrow \eta\mu x = -2$$

$$\Rightarrow \text{αδύνατη}$$

Άρα η αρχική εξίσωση είναι αδύνατη

16.13 7)

Θέτουμε $y = \sigma\upsilon\nu x$ και η εξίσωση γράφεται : $2y^2 - 5y + 2 = 0 \Rightarrow y_{1,2} = \frac{5 \pm \sqrt{9}}{4} \begin{matrix} \nearrow y_1 = 2 \\ \searrow y_2 = \frac{1}{2} \end{matrix}$

$$\Rightarrow y = 2$$

$$\Rightarrow \sigma\upsilon\nu x = 2$$

$$\Rightarrow \text{αδύνατη}$$

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$$\Rightarrow y = \frac{1}{2}$$

$$\Rightarrow \sigma\upsilon\nu x = \frac{1}{2}$$

$$\Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{3}$$

$$\Rightarrow x = 2\kappa\pi \pm \frac{\pi}{3}, \kappa \in \mathbb{Z}$$

16.13 8)

Θέτουμε $y = \sigma\phi x$ και η εξίσωση γράφεται : $\sqrt{3}y^2 + y = 0 \Rightarrow y \cdot (\sqrt{3}y + 1) = 0 \Rightarrow$

$$\Rightarrow y = 0$$

$$\Rightarrow \sigma\phi x = 0$$

$$\Rightarrow \sigma\phi x = \sigma\phi \frac{\pi}{2}$$

$$\Rightarrow x = \kappa\pi + \frac{\pi}{2}, \quad \kappa \in \mathbb{Z}$$

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$$\Rightarrow \sqrt{3}y + 1 = 0$$

$$\Rightarrow \sqrt{3}\sigma\phi x = -1 \Rightarrow \sigma\phi x = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \sigma\phi x = -\sigma\phi \frac{\pi}{3} \Rightarrow \sigma\phi x = \sigma\phi \left(-\frac{\pi}{3}\right)$$

$$\Rightarrow x = \kappa\pi - \frac{\pi}{3}, \quad \kappa \in \mathbb{Z}$$

16.13 9)

Θέτουμε $y = \eta\mu x$ και η εξίσωση γράφεται : $2y^2 - 3y + 1 = 0 \xRightarrow{\Delta=1} y_{1,2} = \frac{3 \pm \sqrt{1}}{4} \begin{matrix} \nearrow y_1 = 1 \\ \searrow y_2 = \frac{1}{2} \end{matrix}$

$$y = 1$$

$$\Rightarrow \eta\mu x = 1$$

$$\Rightarrow \eta\mu x = \eta\mu \frac{\pi}{2}$$

$$\Rightarrow x = 2\kappa\pi + \frac{\pi}{2}, \quad \kappa \in \mathbb{Z}$$

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$$y = \frac{1}{2}$$

$$\Rightarrow \eta\mu x = \frac{1}{2} \Rightarrow \eta\mu x = \eta\mu \frac{\pi}{6}$$

$$\Rightarrow x = 2\kappa\pi + \frac{\pi}{6}$$

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$$x = 2\kappa\pi + \pi - \frac{\pi}{6}, \quad \kappa \in \mathbb{Z}$$

16.13 10)

Θέτουμε $y = \sigma\upsilon\nu x$ και η εξίσωση γράφεται : $2y^2 - y - 1 = 0 \xRightarrow{\Delta=9} y_{1,2} = \frac{1 \pm \sqrt{9}}{4} \begin{matrix} \nearrow y_1 = 1 \\ \searrow y_2 = -\frac{1}{2} \end{matrix}$

$$\Rightarrow y = 1$$

$$\Rightarrow \sigma\upsilon\nu x = 1$$

$$\Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu 0$$

$$\Rightarrow x = 2\kappa\pi, \quad \kappa \in \mathbb{Z}$$

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$$\Rightarrow y = -\frac{1}{2}$$

$$\Rightarrow \sigma\upsilon\nu x = -\frac{1}{2} \Rightarrow \sigma\upsilon\nu x = -\sigma\upsilon\nu \left(\frac{\pi}{3}\right)$$

$$\Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \left(\pi - \frac{\pi}{3}\right)$$

$$\Rightarrow x = 2\kappa\pi \pm \frac{2\pi}{3}, \quad \kappa \in \mathbb{Z}$$

16.13 11)

Θέτουμε $y = \sigma\phi x$ και η εξίσωση γράφεται : $y^2 - (\sqrt{3} + 1)y + \sqrt{3} = 0 \Rightarrow y^2 - \sqrt{3}y - y + \sqrt{3} = 0 \Rightarrow$

$$\Rightarrow y^2 - y + \sqrt{3} - \sqrt{3}y = 0 \Rightarrow y(y - 1) + \sqrt{3}(1 - y) = 0 \Rightarrow (y - 1)(y - \sqrt{3}) = 0 \Rightarrow$$

$$\begin{aligned} &\Rightarrow y = 1 \\ &\Rightarrow \sigma\varphi x = 1 \\ &\Rightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{4} \\ &\Rightarrow x = \kappa\pi + \frac{\pi}{4}, \quad \kappa \in \mathbb{Z} \end{aligned}$$

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$$\begin{aligned} &\Rightarrow y = \sqrt{3} \\ &\Rightarrow \sigma\varphi x = \sqrt{3} \\ &\Rightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{6} \\ &\Rightarrow x = \kappa\pi + \frac{\pi}{6}, \quad \kappa \in \mathbb{Z} \end{aligned}$$

16.13 12)

Θέτουμε $y = \sigma\upsilon\nu x$ και η εξίσωση γράφεται :

$$\begin{aligned} 2y^2 - (\sqrt{2} + 2)y + \sqrt{2} &= 0 \Rightarrow 2y^2 - \sqrt{2}y - 2y + \sqrt{2} = 0 \Rightarrow 2y^2 - 2y + \sqrt{2} - \sqrt{2}y = 0 \Rightarrow \\ &\Rightarrow 2y(y-1) + \sqrt{2}(1-y) = 0 \Rightarrow (2y + \sqrt{2})(y-1) = 0 \Rightarrow \end{aligned}$$

$$\begin{aligned} 2y &= \sqrt{2} \\ &\Rightarrow y = \frac{\sqrt{2}}{2} \\ &\Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{4} \\ &\Rightarrow x = 2\kappa\pi \pm \frac{\pi}{4}, \quad \kappa \in \mathbb{Z} \end{aligned}$$

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$$\begin{aligned} y - 1 &= 0 \\ &\Rightarrow y = 1 \\ &\Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu 0 \\ &\Rightarrow x = 2\kappa\pi, \quad \kappa \in \mathbb{Z} \end{aligned}$$

16.13 13)

Θέτουμε $y = \epsilon\varphi x$ και η εξίσωση γράφεται :

$$\begin{aligned} y^2 - (\sqrt{3} + 1)y + \sqrt{3} &= 0 \Rightarrow y^2 - \sqrt{3}y - y + \sqrt{3} = 0 \Rightarrow y^2 - y + \sqrt{3} - \sqrt{3}y = 0 \Rightarrow \\ &\Rightarrow y(y-1) + \sqrt{3}(1-y) = 0 \Rightarrow (y-1)(y-\sqrt{3}) = 0 \Rightarrow \end{aligned}$$

$$\begin{aligned} y &= 1 \\ &\Rightarrow \epsilon\varphi x = 1 \\ &\Rightarrow \epsilon\varphi x = \epsilon\varphi \frac{\pi}{4} \\ &\Rightarrow x = \kappa\pi + \frac{\pi}{4}, \quad \kappa \in \mathbb{Z} \end{aligned}$$

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$$\begin{aligned} y &= \sqrt{3} \\ &\Rightarrow \epsilon\varphi x = \sqrt{3} \\ &\Rightarrow \epsilon\varphi x = \epsilon\varphi \frac{\pi}{3} \\ &\Rightarrow x = \kappa\pi + \frac{\pi}{3}, \quad \kappa \in \mathbb{Z} \end{aligned}$$

16.13 14)

Θέτουμε $y = \sigma\varphi x$ και η εξίσωση γράφεται : $3y^2 + 2\sqrt{3}y - 3 = 0 \Rightarrow y_{1,2} = \frac{-2\sqrt{3} \pm \sqrt{48}}{4} \begin{matrix} \nearrow y_1 = \frac{\sqrt{3}}{3} \\ \searrow y_2 = -\sqrt{3} \end{matrix}$

$$\begin{aligned} &\Rightarrow y = \frac{\sqrt{3}}{3} \\ &\Rightarrow \sigma\varphi x = \frac{\sqrt{3}}{3} \\ &\Rightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{3} \\ &\Rightarrow x = \kappa\pi + \frac{\pi}{3}, \quad \kappa \in \mathbb{Z} \end{aligned}$$

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$$\begin{aligned} &\Rightarrow y = -\sqrt{3} \\ &\Rightarrow \sigma\varphi x = -\sqrt{3} \\ &\Rightarrow \sigma\varphi x = -\sigma\varphi \frac{\pi}{6} \\ &\Rightarrow x = \kappa\pi - \frac{\pi}{6}, \quad \kappa \in \mathbb{Z} \end{aligned}$$

Θέτουμε $y = \eta \mu x$ και η εξίσωση γράφεται :

$$2y^2 - (4 + \sqrt{2})y + 2\sqrt{2} = 0 \Rightarrow 2y^2 - 4y - \sqrt{2}y + 2\sqrt{2} = 0 \Rightarrow 2y(y - 2) + \sqrt{2}(2 - y) = 0 \Rightarrow$$

$$\Rightarrow (y - 2)(2y - \sqrt{2}) = 0$$

$$\Rightarrow y = 2$$

$\Rightarrow$ αδύνατη

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$$\Rightarrow y = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \eta \mu x = \eta \mu \frac{\pi}{4}$$

$$\Rightarrow x = 2\kappa\pi + \frac{\pi}{4}$$

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$$x = 2\kappa\pi + \pi - \frac{\pi}{4}, \quad \kappa \in \mathbb{Z}$$