## 16.12 1)

**α)** $\eta\mu\left(3x+\frac{\pi}{3}\right)=-\eta\mu\left(x+\frac{\pi}{4}\right)\Rightarrow\eta\mu\left(3x+\frac{\pi}{3}\right)=\eta\mu\left[-\left(x+\frac{\pi}{4}\right)\right]\Rightarrow\eta\mu\left(3x+\frac{\pi}{3}\right)=\eta\mu\left(-x-\frac{\pi}{4}\right)\Rightarrow$

$\Rightarrow 3x+\frac{\pi}{3}=2\kappa\pi-x-\frac{\pi}{4}\Rightarrow$

$\Rightarrow 3x+x=2\kappa\pi-\frac{\pi}{4}+\frac{\pi}{3}\Rightarrow$

$\Rightarrow 4x=2\kappa\pi+\frac{\pi}{12}\Rightarrow x=\frac{\kappa\pi}{2}+\frac{\pi}{48}\ ,\ \kappa\in Z$

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$\Rightarrow 3x+\frac{\pi}{3}=2\kappa\pi+\pi-\left(-x-\frac{\pi}{4}\right)\Rightarrow$

$\Rightarrow 3x+\frac{\pi}{3}=2\kappa\pi+\pi+x+\frac{\pi}{4}\Rightarrow$

$\Rightarrow 3x-x=2\kappa\pi+\pi+\frac{\pi}{4}-\frac{\pi}{3}\Rightarrow 2x=2\kappa\pi+\frac{11\pi}{12}\Rightarrow$

$\Rightarrow x=\kappa\pi+\frac{11\pi}{24}\Rightarrow\ ,\ \kappa\in Z$

**β)** $\sigma\upsilon\nu\left(x+\frac{\pi}{3}\right)=-\sigma\upsilon\nu\left(2x-\frac{\pi}{4}\right)\Rightarrow\sigma\upsilon\nu\left(x+\frac{\pi}{3}\right)=\sigma\upsilon\nu\left[\pi-\left(2x-\frac{\pi}{4}\right)\right]\Rightarrow$

$\Rightarrow\sigma\upsilon\nu\left(x+\frac{\pi}{3}\right)=\sigma\upsilon\nu\left(\pi-2x+\frac{\pi}{4}\right)\Rightarrow\sigma\upsilon\nu\left(x+\frac{\pi}{3}\right)=\sigma\upsilon\nu\left(\frac{5\pi}{4}-2x\right)\Rightarrow$

$\Rightarrow x+\frac{\pi}{3}=2\kappa\pi+\frac{5\pi}{4}-2x\Rightarrow$

$\Rightarrow x+2x=2\kappa\pi+\frac{5\pi}{4}-\frac{\pi}{3}\Rightarrow$

$\Rightarrow 3x=2\kappa\pi+\frac{11\pi}{12}\Rightarrow x=\frac{2\kappa\pi}{3}+\frac{11\pi}{36}\ ,\ \kappa\in Z$

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$\Rightarrow x+\frac{\pi}{3}=2\kappa\pi-\left(\frac{5\pi}{4}-2x\right)\Rightarrow$

$\Rightarrow x+\frac{\pi}{3}=2\kappa\pi-\frac{5\pi}{4}+2x\Rightarrow$

$\Rightarrow x-2x=2\kappa\pi-\frac{5\pi}{4}-\frac{\pi}{3}\Rightarrow$

$\Rightarrow -x=2\kappa\pi-\frac{19\pi}{12}\Rightarrow x=-2\kappa\pi+\frac{19\pi}{12}\ ,\ \kappa\in Z$

## 16.12 2)

$\eta\mu x=-\eta\mu 5x\Rightarrow\eta\mu x=\eta\mu(-5x)\Rightarrow$

$\Rightarrow x=2\kappa\pi-5x$

$\Rightarrow 6x=2\kappa\pi$

$\Rightarrow x=\frac{\kappa\pi}{3}\ ,\ \kappa\in Z$

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$\Rightarrow x=2\kappa\pi+\pi+5x$

$\Rightarrow 4x=-2\kappa\pi-\pi$

$\Rightarrow x=-\frac{\kappa\pi}{2}-\frac{\pi}{4}\ ,\ \kappa\in Z$

## 16.12 3)

$\sigma\upsilon\nu x=-\sigma\upsilon\nu 3x\Rightarrow\sigma\upsilon\nu x=\sigma\upsilon\nu(\pi-5x)\Rightarrow$

$\Rightarrow x=2\kappa\pi+\pi-3x$

$\Rightarrow 4x=2\kappa\pi+\pi$

$\Rightarrow x=\frac{\kappa\pi}{2}+\frac{\pi}{4}\ ,\ \kappa\in Z$

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$\Rightarrow x=2\kappa\pi-\pi+3x$

$\Rightarrow 2x=-2\kappa\pi+\pi$

$\Rightarrow x=-\kappa\pi+\frac{\pi}{2}\ ,\ \kappa\in Z$

## 16.12 4)

$\varepsilon\varphi x=-\varepsilon\varphi 8x\Rightarrow\varepsilon\varphi x=\varepsilon\varphi(-8x)\Rightarrow x=\kappa\pi-8x\Rightarrow 9x=\kappa\pi\Rightarrow x=\frac{\kappa\pi}{9}\ ,\ \kappa\in Z$

## 16.12 5)

$$\sigma\varphi x = -\sigma\varphi 4x \Rightarrow \sigma\varphi x = \sigma\varphi(-4x) \Rightarrow x = \kappa\pi - 4x \Rightarrow 5x = \kappa\pi \Rightarrow x = \frac{\kappa\pi}{5}\ ,\ \kappa \in Z$$

## 16.12 6)

$$\eta\mu\left(x+\frac{\pi}{3}\right) = -\eta\mu\left(x-\frac{\pi}{6}\right) \Rightarrow \eta\mu\left(x+\frac{\pi}{3}\right) = \eta\mu\left(-\left(x-\frac{\pi}{6}\right)\right) \Rightarrow \eta\mu\left(x+\frac{\pi}{3}\right) = \eta\mu\left(\frac{\pi}{6}-x\right) \Rightarrow$$

$$\Rightarrow x+\frac{\pi}{3} = 2\kappa\pi + \frac{\pi}{6} - x$$

$$\Rightarrow 2x = 2\kappa\pi - \frac{\pi}{6}$$

$$\Rightarrow x = \kappa\pi - \frac{\pi}{12}\ ,\ \kappa \in Z$$

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$$\Rightarrow x+\frac{\pi}{3} = 2\kappa\pi + \pi - \frac{\pi}{6} + x$$

$$\Rightarrow \quad \text{αδύνατη}$$

## 16.12 7)

$$\varepsilon\varphi\left(x+\frac{\pi}{4}\right) = -\varepsilon\varphi\left(3x-\frac{\pi}{4}\right) \Rightarrow \varepsilon\varphi\left(x+\frac{\pi}{4}\right) = \varepsilon\varphi\left[-\left(3x-\frac{\pi}{4}\right)\right] \Rightarrow \varepsilon\varphi\left(x+\frac{\pi}{4}\right) = \varepsilon\varphi\left(\frac{\pi}{4}-3x\right) \Rightarrow$$

$$\Rightarrow x+\frac{\pi}{4} = \kappa\pi + \frac{\pi}{4} - 3x \Rightarrow 4x = \kappa\pi \Rightarrow x = \frac{\kappa\pi}{4}\ ,\ \kappa \in Z$$

## 16.12 8)

$$\sigma\upsilon\nu\left(3x+\frac{\pi}{3}\right) = -\sigma\upsilon\nu\left(x+\frac{\pi}{6}\right) \Rightarrow \sigma\upsilon\nu\left(3x+\frac{\pi}{3}\right) = \sigma\upsilon\nu\left[\pi-\left(x+\frac{\pi}{6}\right)\right] \Rightarrow$$

$$\Rightarrow \sigma\upsilon\nu\left(3x+\frac{\pi}{3}\right) = \sigma\upsilon\nu\left(\frac{5\pi}{6}-x\right) \Rightarrow$$

$$\Rightarrow 3x+\frac{\pi}{3} = 2\kappa\pi + \frac{5\pi}{6} - x$$

$$\Rightarrow 4x = 2\kappa\pi + \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\kappa\pi}{2} + \frac{\pi}{8}\ ,\ \kappa \in Z$$

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$$\Rightarrow 3x+\frac{\pi}{3} = 2\kappa\pi - \frac{5\pi}{6} + x$$

$$\Rightarrow 2x = 2\kappa\pi - \frac{7\pi}{6}$$

$$\Rightarrow x = \kappa\pi - \frac{7\pi}{12}\ ,\ \kappa \in Z$$

## 16.12 9)

$$\sigma\varphi\left(2x+\frac{\pi}{4}\right) = -\sigma\varphi\left(\frac{\pi}{6}-4x\right) \Rightarrow \sigma\varphi\left(2x+\frac{\pi}{4}\right) = \sigma\varphi\left[-\left(\frac{\pi}{6}-4x\right)\right] \Rightarrow$$

$$\Rightarrow \sigma\varphi\left(2x+\frac{\pi}{4}\right) = \sigma\varphi\left(4x-\frac{\pi}{6}\right) \Rightarrow 2x+\frac{\pi}{4} = \kappa\pi + 4x - \frac{\pi}{6} \Rightarrow$$

$$\Rightarrow 2x = -\kappa\pi + \frac{5\pi}{12} \Rightarrow x = -\frac{\kappa\pi}{2} + \frac{5\pi}{24}\ ,\ \kappa \in Z$$