16.10 1)

α) $2\sigma\upsilon\nu 4x=\sqrt{3}\Rightarrow \sigma\upsilon\nu 4x=\frac{\sqrt{3}}{2}\Rightarrow \sigma\upsilon\nu 4x=\sigma\upsilon\nu\frac{\pi}{6}\Rightarrow 4x=2\kappa\pi\pm\frac{\pi}{6}\Rightarrow x=\frac{2\kappa\pi}{4}\pm\frac{\frac{\pi}{6}}{4}\Rightarrow$

$\Rightarrow x=\frac{\kappa\pi}{2}\pm\frac{\pi}{24}\quad,\ \kappa\in Z$

β) $2\eta\mu\left(2x-\frac{\pi}{3}\right)=\sqrt{3}\Rightarrow \eta\mu\left(2x-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}\Rightarrow \eta\mu\left(2x-\frac{\pi}{3}\right)=\eta\mu\frac{\pi}{3}\Rightarrow$

$2x-\frac{\pi}{3}=2\kappa\pi+\frac{\pi}{3}\Rightarrow$

$\Rightarrow 2x=2\kappa\pi+\frac{\pi}{3}+\frac{\pi}{3}\Rightarrow 2x=2\kappa\pi+\frac{2\pi}{3}\Rightarrow$

$\Rightarrow \frac{2x}{2}=\frac{2\kappa\pi}{2}+\frac{\frac{2\pi}{3}}{2}\Rightarrow x=\kappa\pi+\frac{\pi}{3}\quad,\ \kappa\in Z$

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$2x-\frac{\pi}{3}=2\kappa\pi+\pi-\frac{\pi}{3}\Rightarrow$

$\Rightarrow 2x=2\kappa\pi+\pi-\frac{\pi}{3}+\frac{\pi}{3}\Rightarrow 2x=2\kappa\pi+\pi\Rightarrow$

$\Rightarrow \frac{2x}{2}=\frac{2\kappa\pi}{2}+\frac{\pi}{2}\Rightarrow x=\kappa\pi+\frac{\pi}{2}\quad,\ \kappa\in Z$

16.10 2)

$\sigma\upsilon\nu 4x=\frac{1}{2}\Rightarrow \sigma\upsilon\nu 4x=\sigma\upsilon\nu\frac{\pi}{3}\Rightarrow 4x=2\kappa\pi\pm\frac{\pi}{3}\Rightarrow x=\frac{2\kappa\pi}{4}\pm\frac{\pi}{12}\Rightarrow x=\frac{\kappa\pi}{2}\pm\frac{\pi}{12}\ ,\ \kappa\in Z$

16.10 3)

$\sigma\varphi 3x=1\Rightarrow \sigma\varphi 3x=\sigma\varphi\frac{\pi}{4}\Rightarrow 3x=\kappa\pi+\frac{\pi}{4}\Rightarrow x=\frac{\kappa\pi}{3}+\frac{\pi}{12}\ ,\ \kappa\in Z$

16.10 4)

$\eta\mu\left(x+\frac{\pi}{4}\right)=-\frac{1}{2}\Rightarrow \eta\mu\left(x+\frac{\pi}{4}\right)=-\eta\mu\frac{\pi}{6}\Rightarrow \eta\mu\left(x+\frac{\pi}{4}\right)=\eta\mu\left(-\frac{\pi}{6}\right)\Rightarrow$

$\Rightarrow x+\frac{\pi}{4}=2\kappa\pi-\frac{\pi}{6}\Rightarrow$

$\Rightarrow x=2\kappa\pi-\frac{5\pi}{12}\ ,\ \kappa\in Z$

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$\Rightarrow x+\frac{\pi}{4}=2\kappa\pi+\pi+\frac{\pi}{6}\Rightarrow$

$\Rightarrow x=2\kappa\pi+\frac{11\pi}{12}\quad,\quad \kappa\in Z$

16.10 5)

$3\varphi\left(3x+\frac{2\pi}{3}\right)=-1\Rightarrow \varepsilon\varphi\left(3x+\frac{2\pi}{3}\right)=-\varepsilon\varphi\frac{\pi}{4}\Rightarrow$

$\Rightarrow \varepsilon\varphi\left(3x+\frac{2\pi}{3}\right)=\varepsilon\varphi\left(-\frac{\pi}{4}\right)\Rightarrow 3x+\frac{2\pi}{3}=\kappa\pi-\frac{\pi}{4}\Rightarrow$

$3x=\kappa\pi-\frac{\pi}{4}-\frac{2\pi}{3}\Rightarrow x=\frac{\kappa\pi}{3}-\frac{\pi}{12}-\frac{2\pi}{9}\Rightarrow$

$\Rightarrow x=\frac{\kappa\pi}{3}-\frac{11\pi}{36}\ ,\ \kappa\in Z$

16.10 6)

$$\sigma\upsilon\nu\left(\frac{\pi}{2}-3x\right)=0\Rightarrow\sigma\upsilon\nu\left(\frac{\pi}{2}-3x\right)=\sigma\upsilon\nu\frac{\pi}{2}\Rightarrow\frac{\pi}{2}-3x=2\kappa\pi\pm\frac{\pi}{2}\Rightarrow$$

$$\frac{\pi}{2}-3x=2\kappa\pi+\frac{\pi}{2}$$
$$\Rightarrow 3x=2\kappa\pi$$
$$\Rightarrow x=\frac{2\kappa\pi}{3}$$
$$\kappa\in Z$$

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$$\frac{\pi}{2}-3x=2\kappa\pi-\frac{\pi}{2}$$
$$\Rightarrow 3x=-2\kappa\pi+\pi$$
$$\Rightarrow x=\frac{-2\kappa\pi}{3}+\frac{\pi}{3}$$
$$\kappa\in Z$$

Συνδυάζοντας τις λύσεις αυτές παίρνουμε $x=\frac{\kappa\pi}{3}$, $\kappa\in Z$

16.10 7)

$$2\sigma\upsilon\nu\left(3x+\frac{\pi}{4}\right)=-1\Rightarrow\sigma\upsilon\nu\left(3x+\frac{\pi}{4}\right)=-\frac{1}{2}\Rightarrow\sigma\upsilon\nu\left(3x+\frac{\pi}{4}\right)=-\sigma\upsilon\nu\frac{\pi}{3}\Rightarrow$$
$$\Rightarrow\sigma\upsilon\nu\left(3x+\frac{\pi}{4}\right)=\sigma\upsilon\nu\left(\frac{2\pi}{3}\right)\Rightarrow 3x+\frac{\pi}{4}=2\kappa\pi\pm\frac{2\pi}{3}\Rightarrow$$

$$3x+\frac{\pi}{4}=2\kappa\pi+\frac{2\pi}{3}$$
$$\Rightarrow 3x=2\kappa\pi+\frac{2\pi}{3}-\frac{\pi}{4}$$
$$\Rightarrow 3x=2\kappa\pi+\frac{5\pi}{12}$$
$$\Rightarrow x=\frac{2\kappa\pi}{3}+\frac{5\pi}{12}$$
$$\kappa\in Z$$

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$$3x+\frac{\pi}{4}=2\kappa\pi-\frac{2\pi}{3}$$
$$\Rightarrow 3x=2\kappa\pi-\frac{2\pi}{3}-\frac{\pi}{4}$$
$$\Rightarrow 3x=2\kappa\pi-\frac{11\pi}{12}$$
$$\Rightarrow x=\frac{2\kappa\pi}{3}-\frac{11\pi}{36}$$
$$\kappa\in Z$$

16.10 8)

$$\sqrt{3}\varepsilon\varphi\left(x-\frac{2\pi}{3}\right)=-1\Rightarrow\varepsilon\varphi\left(x-\frac{2\pi}{3}\right)=-\frac{1}{\sqrt{3}}\Rightarrow\varepsilon\varphi\left(x-\frac{2\pi}{3}\right)=\varepsilon\varphi\left(-\frac{\pi}{6}\right)\Rightarrow$$
$$\Rightarrow x-\frac{2\pi}{3}=\kappa\pi-\frac{\pi}{6}\Rightarrow x=2\kappa\pi-\frac{\pi}{6}+\frac{2\pi}{3}\Rightarrow x=2\kappa\pi+\frac{\pi}{2}\ ,\ \kappa\in Z$$

16.10 9)

$$3\sigma\varphi\left(\frac{\pi}{5}-2x\right)=\sqrt{3}\Rightarrow\sigma\varphi\left(\frac{\pi}{5}-2x\right)=\frac{\sqrt{3}}{3}\Rightarrow\sigma\varphi\left(\frac{\pi}{5}-2x\right)=\sigma\varphi\frac{\pi}{6}\Rightarrow$$
$$\Rightarrow\frac{\pi}{5}-2x=\kappa\pi+\frac{\pi}{3}\Rightarrow 2x=-\kappa\pi+\frac{\pi}{5}-\frac{\pi}{6}\Rightarrow$$
$$\Rightarrow 2x=-\kappa\pi-\frac{\pi}{15}\Rightarrow x=-\kappa\pi-\frac{\pi}{30}$$