

Β ΛΥΚΕΙΟΥ ΑΛΓΕΒΡΑ

14.7 1)

$$\alpha) \quad \alpha' \text{ μέλος} = \eta\mu(\pi - x) + \eta\mu\left(\frac{\pi}{2} - x\right) + \eta\mu(\pi + x) + \eta\mu\left(\frac{3\pi}{2} - x\right)$$

$$\eta\mu(\pi - x) = \eta\mu x, \quad \eta\mu\left(\frac{\pi}{2} - x\right) = \sigma\upsilon\nu x, \quad \eta\mu(\pi + x) = -\eta\mu x, \quad \eta\mu\left(\frac{3\pi}{2} - x\right) = -\sigma\upsilon\nu x$$

$$= \eta\mu x + \sigma\upsilon\nu x - \eta\mu x - \sigma\upsilon\nu x = 0 = \beta' \text{ μέλος}$$

$$\eta\mu(\pi - x) = \eta\mu x$$

$$\epsilon\varphi\left(\frac{3\pi}{2} + x\right) = -\sigma\varphi x$$

$$\eta\mu\left(\frac{\pi}{2} - x\right) = \sigma\upsilon\nu x$$

$$\beta) \quad \frac{\eta\mu(180^\circ - x) \cdot \epsilon\varphi(270^\circ + x)}{\eta\mu(90^\circ - x) \cdot \epsilon\varphi(360^\circ - x)} = \frac{\epsilon\varphi(2\pi - x) = -\epsilon\varphi x}{\eta\mu x \cdot (-\sigma\varphi x)} = \frac{-\eta\mu x \cdot \sigma\varphi x}{\sigma\upsilon\nu x \cdot (-\epsilon\varphi x)} = \frac{-\cancel{\eta\mu x} \cdot \sigma\varphi x}{-\cancel{\sigma\upsilon\nu x} \cdot \frac{\eta\mu x}{\cancel{\sigma\upsilon\nu x}}} = \frac{\cancel{\eta\mu x} \cdot \sigma\varphi x}{\cancel{\eta\mu x}} = \sigma\varphi x$$

14.7 2)

$$\alpha' \text{ μέλος} = \sigma\upsilon\nu(2\pi - x) + \sigma\upsilon\nu(\pi + x) + \eta\mu\left(\frac{3\pi}{2} + x\right) - \eta\mu\left(\frac{3\pi}{2} - x\right) =$$

$$= \sigma\upsilon\nu x + (-\sigma\upsilon\nu x) + (-\sigma\upsilon\nu x) - (-\sigma\upsilon\nu x) = 0 =$$

$$= \beta' \text{ μέλος}$$

14.7 3)

$$\alpha' \text{ μέλος} = \epsilon\varphi(\pi + x) + \epsilon\varphi(2\pi - x) + \epsilon\varphi\left(\frac{\pi}{2} + x\right) - \sigma\varphi(-x) =$$

$$= \epsilon\varphi x + (-\epsilon\varphi x) + (-\sigma\varphi x) - (-\sigma\varphi x) = 0 =$$

$$= \beta' \text{ μέλος}$$

14.7 4)

$$\alpha' \text{ μέλος} = \frac{\eta\mu(\pi + \omega) \cdot \sigma\upsilon\nu\left(\frac{\pi}{2} - \omega\right) \cdot \epsilon\varphi(\pi + \omega)}{\epsilon\varphi(\pi - \omega) \cdot \eta\mu\left(\frac{3\pi}{2} + \omega\right) \cdot \sigma\varphi\left(\frac{\pi}{2} + \omega\right)} = \frac{(-\eta\mu\omega) \cdot \eta\mu\omega \cdot \epsilon\varphi\omega}{(-\epsilon\varphi\omega) \cdot (-\sigma\upsilon\nu\omega) \cdot (-\epsilon\varphi\omega)} =$$

$$= \frac{-\eta\mu^2\omega \cdot \cancel{\eta\mu\omega}}{\cancel{\sigma\upsilon\nu\omega}} = -\eta\mu\omega =$$

$$\frac{\cancel{-\eta\mu\omega}}{\cancel{\sigma\upsilon\nu\omega}} \cdot \cancel{(-\sigma\upsilon\nu\omega)} \cdot \frac{\eta\mu\omega}{\cancel{\sigma\upsilon\nu\omega}} =$$

$$= \beta' \text{ μέλος}$$

14.7 5)

$$\alpha' \text{ μέλος} = \frac{\eta\mu(180^\circ - \omega) \cdot \sigma\varphi(90^\circ - \omega) \cdot \sigma\upsilon\nu(180^\circ - \omega)}{\epsilon\varphi(180^\circ + \omega) \cdot \epsilon\varphi(90^\circ + \omega) \cdot \eta\mu(180^\circ + \omega)} = \frac{\eta\mu\omega \cdot \cancel{\epsilon\varphi\omega} \cdot (-\sigma\upsilon\nu\omega)}{\cancel{\epsilon\varphi\omega} \cdot (-\sigma\varphi\omega) \cdot (-\eta\mu\omega)} =$$

$$= \frac{-\eta\mu\omega \cdot \cancel{\sigma\upsilon\nu\omega}}{\cancel{\sigma\upsilon\nu\omega} \cdot \cancel{\eta\mu\omega}} = -\eta\mu\omega =$$

$$= \beta' \text{ μέλος}$$

14.7 6)

$$\begin{aligned}\alpha' \text{ μέλος} &= \frac{\sigma\upsilon\nu(360^\circ - \omega) \cdot \eta\mu(180^\circ + \omega) \cdot \sigma\varphi(270^\circ - \omega)}{\sigma\upsilon\nu(360^\circ + \omega) \cdot \varepsilon\varphi(180^\circ - \omega) \cdot \sigma\upsilon\nu(90^\circ + \omega)} = \\ &= \frac{\cancel{\sigma\upsilon\nu\omega} \cdot \cancel{(-\eta\mu\omega)} \cdot \cancel{\varepsilon\varphi\omega}}{\cancel{\sigma\upsilon\nu\omega} \cdot \cancel{(-\varepsilon\varphi\omega)} \cdot \cancel{(-\eta\mu\omega)}} = -1 = \\ &= \beta' \text{ μέλος}\end{aligned}$$

14.7 7)

$$\begin{aligned}\alpha' \text{ μέλος} &= \eta\mu(90^\circ - \omega) \cdot \sigma\upsilon\nu(-\omega) + \sigma\upsilon\nu(90^\circ - \omega) \cdot \eta\mu(180^\circ - \omega) = \\ &= \sigma\upsilon\nu\omega \cdot \sigma\upsilon\nu\omega + \eta\mu\omega \cdot \eta\mu\omega = \sigma\upsilon\nu^2\omega + \eta\mu^2\omega = 1 = \beta' \text{ μέλος} \\ &= \beta' \text{ μέλος}\end{aligned}$$

14.7 8)

$$\begin{aligned}\alpha' \text{ μέλος} &= \varepsilon\varphi\left(\frac{\pi}{2} - \omega\right) \cdot \sigma\varphi\left(\frac{3\pi}{2} - \omega\right) + \eta\mu^2(2\pi - \omega) + \sigma\upsilon\nu^2(\pi + \omega) = \\ &= \varepsilon\varphi\left(\frac{\pi}{2} - \omega\right) \cdot \sigma\varphi\left(\pi + \frac{\pi}{2} - \omega\right) + (-\eta\mu\omega)^2 + (-\sigma\upsilon\nu\omega)^2 = \\ &= \varepsilon\varphi\left(\frac{\pi}{2} - \omega\right) \cdot \sigma\varphi\left(\frac{\pi}{2} - \omega\right) + \eta\mu^2\omega + \sigma\upsilon\nu^2\omega = 1 + 1 = 2 = \\ &= \beta' \text{ μέλος}\end{aligned}$$