

Β ΛΥΚΕΙΟΥ ΑΛΓΕΒΡΑ

13.8 1)

$$\begin{aligned}
 \alpha) \quad \alpha' \text{ μέλος} &= \sin^2 x \sin^2 y - \eta\mu^2 x \eta\mu^2 y \stackrel{\substack{\eta\mu^2 x = 1 - \sin^2 x \\ \eta\mu^2 y = 1 - \sin^2 y}}{=} \sin^2 x \sin^2 y - (1 - \sin^2 x)(1 - \sin^2 y) = \\
 &= \sin^2 x \sin^2 y - (1 - \sin^2 y - \sin^2 x + \sin^2 x \sin^2 y) = \\
 &= \cancel{\sin^2 x \sin^2 y} - 1 + \sin^2 y + \sin^2 x - \cancel{\sin^2 x \sin^2 y} = \sin^2 y + \sin^2 x - 1 = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

$$\begin{aligned}
 \beta) \quad \alpha' \text{ μέλος} &= \frac{1 + \varepsilon\varphi^2 \omega}{\varepsilon\varphi^2 \omega} \stackrel{\varepsilon\varphi^2 \omega = \frac{\eta\mu^2 \omega}{\sin^2 \omega}}{=} \frac{1 + \frac{\eta\mu^2 \omega}{\sin^2 \omega}}{\frac{\eta\mu^2 \omega}{\sin^2 \omega}} = \frac{\frac{\sin^2 \omega + \eta\mu^2 \omega}{\cancel{\sin^2 \omega}}}{\frac{\eta\mu^2 \omega}{\cancel{\sin^2 \omega}}} = \frac{\sin^2 \omega + \eta\mu^2 \omega}{\eta\mu^2 \omega} = \frac{1}{\eta\mu^2 \omega} = \beta' \text{ μέλος}
 \end{aligned}$$

13.8 2)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \eta\mu\omega \sin\omega (\varepsilon\varphi\omega + \sigma\varphi\omega) = \eta\mu\omega \sin\omega \left(\frac{\eta\mu\omega}{\sin\omega} + \frac{\sin\omega}{\eta\mu\omega} \right) = \\
 &= \frac{\eta\mu^2 \omega \sin\omega}{\sin\omega} + \frac{\eta\mu\omega \sin^2 \omega}{\eta\mu\omega} = \eta\mu^2 \omega + \sin^2 \omega = 1 = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

13.8 3)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \eta\mu^4 \omega + \eta\mu^2 \omega \sin^2 \omega = \eta\mu^2 \omega (\eta\mu^2 \omega + \sin^2 \omega) = \\
 &= \eta\mu^2 \omega = 1 - \sin^2 \omega = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

13.8 4)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \eta\mu^2 \alpha \sin^2 \beta + \eta\mu^2 \alpha \eta\mu^2 \alpha + \sin^2 \alpha = \\
 &= \eta\mu^2 \alpha (\sin^2 \beta + \eta\mu^2 \beta) + \sin^2 \alpha = \eta\mu^2 \alpha + \sin^2 \alpha = 1 = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

13.8 5)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \sin^2 \alpha \eta\mu^2 \beta + \sin^2 \alpha \sin^2 \beta + \eta\mu^2 \alpha = \\
 &= \sin^2 \alpha (\eta\mu^2 \beta + \sin^2 \beta) + \eta\mu^2 \alpha = \sin^2 \alpha + \eta\mu^2 \alpha = 1 = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

13.8 6)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \eta\mu^4 \omega + \sin^4 \omega = (\eta\mu^2 \omega + \sin^2 \omega) - 2\sin^2 \omega \eta\mu^2 \omega = \\
 &= 1 - 2\eta\mu^2 \omega \cdot \sin^2 \omega = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

13.8 7)

Έχουμε ισοδύναμα :

$$\eta\mu^2 \alpha - \sin^2 \beta = \eta\mu^2 \beta - \sin^2 \alpha \Leftrightarrow \eta\mu^2 \alpha + \sin^2 \alpha = \eta\mu^2 \beta + \sin^2 \beta \Leftrightarrow$$

$$\Leftrightarrow 1 = 1 \quad \text{που ισχύει}$$

13.8 8)

$$\begin{aligned}\alpha' \text{ μέλος} &= \begin{vmatrix} \frac{\sigma\upsilon\nu\omega}{\sqrt{2}} & \frac{-\eta\mu\omega}{\sqrt{2}} \\ \frac{\eta\mu\omega}{\sqrt{2}} & \frac{\sigma\upsilon\nu\omega}{\sqrt{2}} \end{vmatrix} = \frac{\sigma\upsilon\nu\omega}{\sqrt{2}} \cdot \frac{\sigma\upsilon\nu\omega}{\sqrt{2}} - \left(\frac{-\eta\mu\omega}{\sqrt{2}} \right) \cdot \frac{\eta\mu\omega}{\sqrt{2}} = \\ &= \frac{\sigma\upsilon\nu^2\omega}{2} + \frac{\eta\mu^2\omega}{2} = \frac{\eta\mu^2\omega + \sigma\upsilon\nu^2\omega}{2} = \frac{1}{2} = \\ &= \beta' \text{ μέλος}\end{aligned}$$

13.8 9)

$$\begin{aligned}\alpha' \text{ μέλος} &= \begin{vmatrix} \eta\mu\omega - \sigma\upsilon\nu\omega & \eta\mu\omega + \sigma\upsilon\nu\omega \\ -\eta\mu\omega - \sigma\upsilon\nu\omega & \eta\mu\omega - \sigma\upsilon\nu\omega \end{vmatrix} = \\ &= (\eta\mu\omega - \sigma\upsilon\nu\omega)^2 - (\eta\mu\omega + \sigma\upsilon\nu\omega)(-\eta\mu\omega - \sigma\upsilon\nu\omega) = \\ &= \eta\mu^2\omega - 2\eta\mu\omega\sigma\upsilon\nu\omega + \sigma\upsilon\nu^2\omega - [-\eta\mu^2\omega - \eta\mu\omega\sigma\upsilon\nu\omega - \eta\mu\omega\sigma\upsilon\nu\omega - \sigma\upsilon\nu^2\omega] = \\ &= 1 - 2\eta\mu\omega\sigma\upsilon\nu\omega - [-1 - 2\eta\mu\omega\sigma\upsilon\nu\omega] = \\ &= 1 - 2\eta\mu\omega\sigma\upsilon\nu\omega + 1 + 2\eta\mu\omega\sigma\upsilon\nu\omega = 2 = \\ &= \beta' \text{ μέλος}\end{aligned}$$

13.8 10)

$$\begin{aligned}\alpha' \text{ μέλος} &= \varepsilon\phi\omega(1 - \sigma\phi^2\omega) + \sigma\phi\omega(1 - \varepsilon\phi^2\omega) = \\ &= \frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} \left(1 - \frac{\sigma\upsilon\nu^2\omega}{\eta\mu^2\omega} \right) + \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega} \left(1 - \frac{\eta\mu^2\omega}{\sigma\upsilon\nu^2\omega} \right) = \\ &= \frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} - \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega} + \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega} - \frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} = 0 = \\ &= \beta' \text{ μέλος}\end{aligned}$$

13.8 11)

$$\begin{aligned}\alpha' \text{ μέλος} &= \frac{1}{1 - \eta\mu\omega} + \frac{1}{1 + \eta\mu\omega} = \frac{1 + \eta\mu\omega}{(1 - \eta\mu\omega)(1 + \eta\mu\omega)} + \frac{1 - \eta\mu\omega}{(1 - \eta\mu\omega)(1 + \eta\mu\omega)} = \\ &= \frac{2}{(1 - \eta\mu\omega)(1 + \eta\mu\omega)} = \frac{2}{1^2 - \eta\mu^2\omega} = \frac{2}{\sigma\upsilon\nu^2\omega} = \\ &= \beta' \text{ μέλος}\end{aligned}$$

13.8 12)

$$\begin{aligned}\alpha' \text{ μέλος} &= \frac{1}{\varepsilon\phi\omega - 1} = \frac{1}{\frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} - 1} = \frac{1}{\frac{\eta\mu\omega - \sigma\upsilon\nu\omega}{\sigma\upsilon\nu\omega}} = \\ &= \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega - \sigma\upsilon\nu\omega} = \\ &= \beta' \text{ μέλος}\end{aligned}$$

13.8 13)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \frac{\varepsilon\varphi\omega + 1}{\varepsilon\varphi\omega - 1} = \frac{\frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} + 1}{\frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} - 1} = \frac{\frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} + \frac{\sigma\upsilon\nu\omega}{\sigma\upsilon\nu\omega}}{\frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} - \frac{\sigma\upsilon\nu\omega}{\sigma\upsilon\nu\omega}} = \\
 &= \frac{\frac{\eta\mu\omega + \sigma\upsilon\nu\omega}{\sigma\upsilon\nu\omega}}{\frac{\eta\mu\omega - \sigma\upsilon\nu\omega}{\sigma\upsilon\nu\omega}} = \frac{\eta\mu\omega + \sigma\upsilon\nu\omega}{\eta\mu\omega - \sigma\upsilon\nu\omega} = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

13.8 14)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \frac{1 - \varepsilon\varphi^2\omega}{\varepsilon\varphi\omega} = \frac{1 - \frac{\eta\mu^2\omega}{\sigma\upsilon\nu^2\omega}}{\frac{\eta\mu\omega}{\sigma\upsilon\nu\omega}} = \frac{\frac{\sigma\upsilon\nu^2\omega}{\sigma\upsilon\nu^2\omega} - \frac{\eta\mu^2\omega}{\sigma\upsilon\nu^2\omega}}{\frac{\eta\mu\omega}{\sigma\upsilon\nu\omega}} = \\
 &= \frac{\frac{\sigma\upsilon\nu^2\omega - \eta\mu^2\omega}{\sigma\upsilon\nu^2\omega}}{\frac{\eta\mu\omega}{\sigma\upsilon\nu\omega}} = \frac{(\sigma\upsilon\nu^2\omega - \eta\mu^2\omega) \cdot \sigma\upsilon\nu\omega}{\sigma\upsilon\nu^2\omega \cdot \eta\mu\omega} = \frac{\sigma\upsilon\nu^2\omega - \eta\mu^2\omega}{\eta\mu\omega \cdot \sigma\upsilon\nu\omega} = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

13.8 15)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \frac{\eta\mu\omega}{1 - \sigma\varphi\omega} + \frac{\sigma\upsilon\nu\omega}{1 - \varepsilon\varphi\omega} = \frac{\eta\mu\omega}{1 - \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega}} + \frac{\sigma\upsilon\nu\omega}{1 - \frac{\eta\mu\omega}{\sigma\upsilon\nu\omega}} = \\
 &= \frac{\frac{\eta\mu\omega}{\eta\mu\omega - \sigma\upsilon\nu\omega}}{\frac{\eta\mu\omega}{\eta\mu\omega}} + \frac{\frac{\sigma\upsilon\nu\omega}{\sigma\upsilon\nu\omega - \eta\mu\omega}}{\frac{\sigma\upsilon\nu\omega}{\sigma\upsilon\nu\omega}} = \frac{\eta\mu^2\omega}{\eta\mu\omega - \sigma\upsilon\nu\omega} + \frac{\sigma\upsilon\nu^2\omega}{\sigma\upsilon\nu\omega - \eta\mu\omega} = \\
 &= \frac{\eta\mu^2\omega}{\eta\mu\omega - \sigma\upsilon\nu\omega} - \frac{\sigma\upsilon\nu^2\omega}{\eta\mu\omega - \sigma\upsilon\nu\omega} = \frac{\eta\mu^2\omega - \sigma\upsilon\nu^2\omega}{\eta\mu\omega - \sigma\upsilon\nu\omega} = \\
 &= \frac{(\eta\mu\omega - \sigma\upsilon\nu\omega)(\eta\mu\omega + \sigma\upsilon\nu\omega)}{\eta\mu\omega - \sigma\upsilon\nu\omega} = \eta\mu\omega + \sigma\upsilon\nu\omega = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

13.8 16)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \varepsilon\varphi\omega + \frac{\sigma\upsilon\nu\omega}{1 + \eta\mu\omega} = \frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} + \frac{\sigma\upsilon\nu\omega}{1 + \eta\mu\omega} = \\
 &= \frac{\frac{\eta\mu\omega(1 + \eta\mu\omega)}{\sigma\upsilon\nu\omega(1 + \eta\mu\omega)}}{\frac{\sigma\upsilon\nu\omega(1 + \eta\mu\omega)}{\sigma\upsilon\nu\omega(1 + \eta\mu\omega)}} + \frac{\frac{\sigma\upsilon\nu^2\omega}{\sigma\upsilon\nu\omega(1 + \eta\mu\omega)}}{\frac{\sigma\upsilon\nu\omega(1 + \eta\mu\omega)}{\sigma\upsilon\nu\omega(1 + \eta\mu\omega)}} = \frac{\eta\mu\omega + \eta\mu^2\omega + \sigma\upsilon\nu^2\omega}{\sigma\upsilon\nu\omega(1 + \eta\mu\omega)} = \\
 &= \frac{1 + \eta\mu\omega}{\sigma\upsilon\nu\omega(1 + \eta\mu\omega)} = \frac{1}{\sigma\upsilon\nu\omega} = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

13.8 17)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= 1 - \frac{\eta\mu^2\omega}{1 + \sigma\upsilon\nu\omega} = \frac{1 + \sigma\upsilon\nu\omega}{1 + \sigma\upsilon\nu\omega} - \frac{\eta\mu^2\omega}{1 + \sigma\upsilon\nu\omega} = \\
 &= \frac{1 - \eta\mu^2\omega + \sigma\upsilon\nu\omega}{1 + \sigma\upsilon\nu\omega} = \frac{\sigma\upsilon\nu^2\omega + \sigma\upsilon\nu\omega}{1 + \sigma\upsilon\nu\omega} = \frac{\sigma\upsilon\nu\omega(1 + \sigma\upsilon\nu\omega)}{1 + \sigma\upsilon\nu\omega} =
 \end{aligned}$$

$$= \sigma\omega =$$

$$= \beta' \text{ μέλος}$$

13.8 18)

$$\alpha' \text{ μέλος} = 1 - \frac{\sigma\omega^2}{1 + \eta\omega} = \frac{1 + \eta\omega}{1 + \eta\omega} - \frac{\sigma\omega^2}{1 + \eta\omega} =$$

$$= \frac{1 - \sigma\omega^2 + \eta\omega}{1 + \eta\omega} = \frac{\eta\omega^2 + \eta\omega}{1 + \eta\omega} = \frac{\eta\omega(1 + \eta\omega)}{1 + \eta\omega} = \eta\omega =$$

$$= \beta' \text{ μέλος}$$

13.8 19)

$$\alpha' \text{ μέλος} = \frac{\epsilon\phi^2\omega - 1}{\epsilon\phi^2\omega + 1} = \frac{\frac{\eta\mu^2\omega}{\sigma\omega^2} - 1}{\frac{\eta\mu^2\omega}{\sigma\omega^2} + 1} =$$

$$= \frac{\eta\mu^2\omega - \sigma\omega^2}{\eta\mu^2\omega + \sigma\omega^2} = \frac{\eta\mu^2\omega - \sigma\omega^2}{\sigma\omega^2} = (\eta\mu^2\omega - \sigma\omega^2)(\eta\mu^2\omega + \sigma\omega^2) =$$

$$= \frac{\sigma\omega^2}{\eta\mu^2\omega + \sigma\omega^2} = \frac{\sigma\omega^2}{\sigma\omega^2} =$$

$$= \eta\mu^4\omega - \sigma\omega^4\omega =$$

$$= \beta' \text{ μέλος}$$

13.8 20)

$$\alpha' \text{ μέλος} = \frac{\sigma\omega^2}{1 + \epsilon\phi^2\omega} + \frac{\eta\mu^2\omega}{1 + \sigma\phi\omega} = \frac{\sigma\omega^2}{1 + \frac{\eta\omega}{\sigma\omega}} + \frac{\eta\mu^2\omega}{1 + \frac{\sigma\omega}{\eta\omega}} =$$

$$= \frac{\sigma\omega^2}{\sigma\omega + \eta\omega} + \frac{\eta\mu^2\omega}{\eta\omega + \sigma\omega} = \frac{\sigma\omega^3}{\eta\omega + \sigma\omega} + \frac{\eta\mu^3\omega}{\eta\omega + \sigma\omega} =$$

$$= \frac{\sigma\omega^3 + \eta\mu^3\omega}{\eta\omega + \sigma\omega} = \frac{(\sigma\omega + \eta\omega)(\sigma\omega^2 - \eta\mu\omega\sigma\omega + \eta\mu^2\omega)}{\sigma\omega + \eta\omega} =$$

$$= \sigma\omega^2 - \eta\mu\omega\sigma\omega + \eta\mu^2\omega = 1 - \eta\mu\omega \cdot \sigma\omega =$$

$$= \beta' \text{ μέλος}$$

13.8 21)

$$\alpha' \text{ μέλος} = \frac{\epsilon\phi^3\omega}{1 + \epsilon\phi^2\omega} + \frac{\sigma\phi^3\omega}{1 + \sigma\phi^2\omega} = \frac{\frac{\eta\mu^3\omega}{\sigma\omega^3}}{1 + \frac{\eta\mu^2\omega}{\sigma\omega^2}} + \frac{\frac{\sigma\omega^3}{\eta\mu^3\omega}}{1 + \frac{\sigma\omega^2}{\eta\mu^2\omega}} =$$

$$= \frac{\frac{\eta\mu^3\omega}{\sigma\omega^3}}{\frac{\sigma\omega^2 + \eta\mu^2\omega}{\sigma\omega^2}} + \frac{\frac{\sigma\omega^3}{\eta\mu^3\omega}}{\frac{\eta\mu^2\omega + \sigma\omega^2}{\eta\mu^2\omega}} = \frac{\eta\mu^3\omega \cdot \sigma\omega^2}{\sigma\omega^3} + \frac{\sigma\omega^3 \cdot \eta\mu^2\omega}{\eta\mu^3\omega} =$$

$$= \frac{\eta\mu^3\omega}{\sigma\omega} + \frac{\sigma\omega^3}{\eta\mu\omega} = \frac{\eta\mu^4\omega + \sigma\omega^4\omega}{\eta\mu\omega \cdot \sigma\omega} = \frac{(\eta\mu^2\omega + \sigma\omega^2)\omega - 2\eta\mu^2\omega \cdot \sigma\omega^2\omega}{\eta\mu\omega \cdot \sigma\omega} =$$

$$= \frac{1 - 2\eta\mu^2\omega \cdot \sigma\upsilon\nu^2\omega}{\eta\mu\omega \cdot \sigma\upsilon\nu\omega} =$$

$$= \beta' \text{ μέλος}$$

13.8 22)

$$\alpha' \text{ μέλος} = \frac{\eta\mu^3\omega + \sigma\upsilon\nu^3\omega}{\eta\mu\omega + \sigma\upsilon\nu\omega} = \frac{(\eta\mu\omega + \sigma\upsilon\nu\omega)(\eta\mu^2\omega - \eta\mu\omega \cdot \sigma\upsilon\nu\omega + \sigma\upsilon\nu^2\omega)}{\eta\mu\omega + \sigma\upsilon\nu\omega} =$$

$$= \eta\mu^2\omega - \eta\mu\omega \cdot \sigma\upsilon\nu\omega + \sigma\upsilon\nu^2\omega = 1 - \eta\mu\omega \cdot \sigma\upsilon\nu\omega =$$

$$= \beta' \text{ μέλος}$$

13.8 23)

Έχουμε ισοδύναμα :

$$\frac{(1 - \eta\mu\omega \cdot \sigma\upsilon\nu\omega)(\eta\mu^2\omega - \sigma\upsilon\nu^2\omega)}{\sigma\upsilon\nu\omega \left(\frac{1}{\sigma\upsilon\nu\omega} - \frac{1}{\eta\mu\omega} \right) (\eta\mu^3\omega + \sigma\upsilon\nu^3\omega)} = \eta\mu\omega \Leftrightarrow$$

$$\Leftrightarrow (1 - \eta\mu\omega \cdot \sigma\upsilon\nu\omega)(\eta\mu^2\omega - \sigma\upsilon\nu^2\omega) = \eta\mu\omega \cdot \sigma\upsilon\nu\omega \left(\frac{1}{\sigma\upsilon\nu\omega} - \frac{1}{\eta\mu\omega} \right) (\eta\mu^3\omega + \sigma\upsilon\nu^3\omega) \Leftrightarrow$$

$$\Leftrightarrow (1 - \eta\mu\omega \cdot \sigma\upsilon\nu\omega)(\eta\mu^2\omega - \sigma\upsilon\nu^2\omega) = \eta\mu\omega \left(1 - \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega} \right) (\eta\mu^3\omega + \sigma\upsilon\nu^3\omega) \Leftrightarrow$$

$$\Leftrightarrow (1 - \eta\mu\omega \cdot \sigma\upsilon\nu\omega)(\eta\mu^2\omega - \sigma\upsilon\nu^2\omega) = (\eta\mu\omega - \sigma\upsilon\nu\omega)(\eta\mu^3\omega + \sigma\upsilon\nu^3\omega) \Leftrightarrow$$

$$\Leftrightarrow (1 - \eta\mu\omega \cdot \sigma\upsilon\nu\omega)(\eta\mu\omega - \sigma\upsilon\nu\omega)(\eta\mu\omega + \sigma\upsilon\nu\omega) = (\eta\mu\omega - \sigma\upsilon\nu\omega)(\eta\mu^3\omega + \sigma\upsilon\nu^3\omega) \Leftrightarrow$$

$$\Leftrightarrow (1 - \eta\mu\omega \cdot \sigma\upsilon\nu\omega)(\eta\mu\omega + \sigma\upsilon\nu\omega) = \eta\mu^3\omega + \sigma\upsilon\nu^3\omega \Leftrightarrow$$

$$\Leftrightarrow (\eta\mu^2\omega + \sigma\upsilon\nu^2\omega - \eta\mu\omega \cdot \sigma\upsilon\nu\omega)(\eta\mu\omega + \sigma\upsilon\nu\omega) = \eta\mu^3\omega + \sigma\upsilon\nu^3\omega$$

που ισχύει , λόγω της ταυτότητας $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$