

13.13 1)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \frac{\sqrt{1+\eta\mu\omega} + \sqrt{1-\eta\mu\omega}}{\sqrt{1+\eta\mu\omega} - \sqrt{1-\eta\mu\omega}} \overset{\substack{\text{πολλαπλασιάζουμε αριθμητή και} \\ \text{παρονομαστή με την συζυγή} \\ \text{παράσταση του παρονομαστή}}}{=} \\
 &= \frac{(\sqrt{1+\eta\mu\omega} + \sqrt{1-\eta\mu\omega})(\sqrt{1+\eta\mu\omega} + \sqrt{1-\eta\mu\omega})}{(\sqrt{1+\eta\mu\omega} - \sqrt{1-\eta\mu\omega})(\sqrt{1+\eta\mu\omega} + \sqrt{1-\eta\mu\omega})} = \frac{(\sqrt{1+\eta\mu\omega} + \sqrt{1-\eta\mu\omega})^2}{\sqrt{1+\eta\mu\omega}^2 - \sqrt{1-\eta\mu\omega}^2} = \\
 &= \frac{\sqrt{1+\eta\mu\omega}^2 + \sqrt{1-\eta\mu\omega}^2 + 2\sqrt{1+\eta\mu\omega}\sqrt{1-\eta\mu\omega}}{1+\eta\mu\omega - (1-\eta\mu\omega)} = \frac{1+\cancel{\eta\mu\omega} + 1-\cancel{\eta\mu\omega} + 2\sqrt{(1+\eta\mu\omega)(1-\eta\mu\omega)}}{1+\eta\mu\omega - 1 + \eta\mu\omega} = \\
 &= \frac{2 + 2\sqrt{1-\eta\mu^2\omega}}{2\eta\mu\omega} \overset{1-\eta\mu^2\omega = \sigma\upsilon\nu^2\omega}{=} \frac{2 + 2\sqrt{\sigma\upsilon\nu^2\omega}}{2\eta\mu\omega} = \frac{\cancel{2}(1+|\sigma\upsilon\nu\omega|)}{\cancel{2}\eta\mu\omega} \overset{0 < \omega < \frac{\pi}{2} \Rightarrow \sigma\upsilon\nu\omega > 0}{=} \frac{1+\sigma\upsilon\nu\omega}{\eta\mu\omega} = \beta' \text{ μέλος}
 \end{aligned}$$

ακόμη

$$\begin{aligned}
 \beta' \text{ μέλος} &= \frac{1+\sigma\upsilon\nu\omega}{\eta\mu\omega} \overset{\substack{\text{πολλαπλασιάζουμε αριθμητή και} \\ \text{παρονομαστή με την συζυγή} \\ \text{παράσταση του } 1+\sigma\upsilon\nu\omega}}{=} \frac{(1+\sigma\upsilon\nu\omega)(1-\sigma\upsilon\nu\omega)}{\eta\mu\omega(1-\sigma\upsilon\nu\omega)} = \frac{1-\sigma\upsilon\nu^2\omega}{\eta\mu\omega(1-\sigma\upsilon\nu\omega)} = \\
 &= \frac{\eta\mu^2\omega}{\cancel{\eta\mu\omega}(1-\sigma\upsilon\nu\omega)} = \frac{\eta\mu\omega}{1-\sigma\upsilon\nu\omega} = \gamma' \text{ μέλος}
 \end{aligned}$$

Επομένως τελικά $\alpha' \text{ μέλος} = \beta' \text{ μέλος} = \gamma' \text{ μέλος}$

13.13 2)

$$\begin{aligned}
 \alpha' \text{ μέλος} &= \frac{\sqrt{1+\sigma\upsilon\nu\omega} - \sqrt{1-\sigma\upsilon\nu\omega}}{\sqrt{1+\sigma\upsilon\nu\omega} + \sqrt{1-\sigma\upsilon\nu\omega}} = \\
 &= \frac{(\sqrt{1+\sigma\upsilon\nu\omega} - \sqrt{1-\sigma\upsilon\nu\omega})(\sqrt{1+\sigma\upsilon\nu\omega} - \sqrt{1-\sigma\upsilon\nu\omega})}{(\sqrt{1+\sigma\upsilon\nu\omega} + \sqrt{1-\sigma\upsilon\nu\omega})(\sqrt{1+\sigma\upsilon\nu\omega} - \sqrt{1-\sigma\upsilon\nu\omega})} = \\
 &= \frac{(\sqrt{1+\sigma\upsilon\nu\omega} - \sqrt{1-\sigma\upsilon\nu\omega})^2}{\sqrt{1+\sigma\upsilon\nu\omega}^2 - \sqrt{1-\sigma\upsilon\nu\omega}^2} = \frac{1+\sigma\upsilon\nu\omega - 2\sqrt{1+\sigma\upsilon\nu\omega}\sqrt{1-\sigma\upsilon\nu\omega} + 1-\sigma\upsilon\nu\omega}{1+\sigma\upsilon\nu\omega - (1-\sigma\upsilon\nu\omega)} = \\
 &= \frac{2 - 2\sqrt{(1+\sigma\upsilon\nu\omega)(1-\sigma\upsilon\nu\omega)}}{2\sigma\upsilon\nu\omega} = \frac{1-\sqrt{1-\sigma\upsilon\nu^2\omega}}{\sigma\upsilon\nu\omega} = \\
 &= \frac{1-\sqrt{\eta\mu^2\omega}}{\sigma\upsilon\nu\omega} \overset{0 < \omega < \frac{\pi}{2}}{=} \frac{1-\eta\mu\omega}{\sigma\upsilon\nu\omega} = \\
 &= \beta' \text{ μέλος}
 \end{aligned}$$

Ακόμη

$$\begin{aligned}
 \beta' \text{ μέλος} &= \frac{1-\eta\mu\omega}{\sigma\upsilon\nu\omega} = \frac{(1-\eta\mu\omega)(1+\eta\mu\omega)}{\sigma\upsilon\nu\omega(1+\eta\mu\omega)} = \frac{1-\eta\mu^2\omega}{\sigma\upsilon\nu\omega(1+\eta\mu\omega)} = \\
 &= \frac{\sigma\upsilon\nu^2\omega}{\sigma\upsilon\nu\omega(1+\eta\mu\omega)} = \frac{\sigma\upsilon\nu\omega}{1+\eta\mu\omega} = \\
 &= \gamma' \text{ μέλος}
 \end{aligned}$$

13.13 3)

$$\begin{aligned}
\alpha' \text{ μέλος} &= \sqrt{\frac{1+\sigma\eta\nu\omega}{1-\sigma\eta\nu\omega}} + \sqrt{\frac{1-\sigma\eta\nu\omega}{1+\sigma\eta\nu\omega}} = \\
&= \frac{\sqrt{1+\sigma\eta\nu\omega}\sqrt{1+\sigma\eta\nu\omega}}{\sqrt{1-\sigma\eta\nu\omega}\sqrt{1+\sigma\eta\nu\omega}} + \frac{\sqrt{1-\sigma\eta\nu\omega}\sqrt{1-\sigma\eta\nu\omega}}{\sqrt{1+\sigma\eta\nu\omega}\sqrt{1-\sigma\eta\nu\omega}} = \\
&= \frac{(1+\sigma\eta\nu\omega)}{\sqrt{(1-\sigma\eta\nu\omega)(1+\sigma\eta\nu\omega)}} + \frac{(1-\sigma\eta\nu\omega)}{\sqrt{(1-\sigma\eta\nu\omega)(1+\sigma\eta\nu\omega)}} = \\
&= \frac{1+\sigma\eta\nu\omega}{\sqrt{1-\sigma\eta\nu^2\omega}} + \frac{1-\sigma\eta\nu\omega}{\sqrt{1-\sigma\eta\nu^2\omega}} = \frac{2}{\sqrt{\eta\mu^2\omega}} \stackrel{0<\omega<\pi}{=} \frac{2}{\eta\mu\omega} = \\
&= \beta' \text{ μέλος}
\end{aligned}$$

13.13 4)

$$\begin{aligned}
\alpha' \text{ μέλος} &= \sqrt{\frac{1+\eta\mu\omega}{1-\eta\mu\omega}} + \sqrt{\frac{1-\eta\mu\omega}{1+\eta\mu\omega}} = \\
&= \frac{\sqrt{1+\eta\mu\omega}\sqrt{1+\eta\mu\omega}}{\sqrt{1+\eta\mu\omega}\sqrt{1-\eta\mu\omega}} + \frac{\sqrt{1-\eta\mu\omega}\sqrt{1-\eta\mu\omega}}{\sqrt{1+\eta\mu\omega}\sqrt{1-\eta\mu\omega}} = \\
&= \frac{1+\eta\mu\omega}{\sqrt{1-\eta\mu^2\omega}} + \frac{1-\eta\mu\omega}{\sqrt{1-\eta\mu^2\omega}} = \frac{2}{\sqrt{\sigma\eta\nu^2\omega}} \stackrel{-\frac{\pi}{2}<\omega<\frac{\pi}{2}}{=} \frac{2}{\sigma\eta\nu\omega} = \\
&= \beta' \text{ μέλος}
\end{aligned}$$

13.13 5)

$$\begin{aligned}
\alpha' \text{ μέλος} &= \sqrt{\frac{1+\sigma\eta\nu\omega}{1-\sigma\eta\nu\omega}} - \sqrt{\frac{1-\sigma\eta\nu\omega}{1+\sigma\eta\nu\omega}} = \\
&= \frac{\sqrt{1+\sigma\eta\nu\omega}\sqrt{1+\sigma\eta\nu\omega}}{\sqrt{1-\sigma\eta\nu\omega}\sqrt{1+\sigma\eta\nu\omega}} - \frac{\sqrt{1-\sigma\eta\nu\omega}\sqrt{1-\sigma\eta\nu\omega}}{\sqrt{1+\sigma\eta\nu\omega}\sqrt{1-\sigma\eta\nu\omega}} = \\
&= \frac{1+\sigma\eta\nu\omega}{\sqrt{1-\sigma\eta\nu^2\omega}} - \frac{1-\sigma\eta\nu\omega}{\sqrt{1-\sigma\eta\nu^2\omega}} = \frac{2}{\sqrt{\sigma\eta\nu^2\omega}} = \frac{2\sigma\eta\nu\omega}{\sqrt{\eta\mu^2\omega}} \stackrel{0<\omega<\pi}{=} \frac{2\sigma\eta\nu\omega}{\eta\mu\omega} = 2\sigma\varphi\omega = \\
&= \beta' \text{ μέλος}
\end{aligned}$$