

13.12 1)

α) $\eta\mu\omega + \sigma\upsilon\nu\omega = \lambda \Rightarrow (\eta\mu\omega + \sigma\upsilon\nu\omega)^2 = \lambda^2 \Rightarrow \eta\mu^2\omega + 2\eta\mu\omega\sigma\upsilon\nu\omega + \sigma\upsilon\nu^2\omega = \lambda^2 \Rightarrow$

$$\Rightarrow 1 + 2\eta\mu\omega\sigma\upsilon\nu\omega = \lambda^2 \Rightarrow 2\eta\mu\omega\sigma\upsilon\nu\omega = \lambda^2 - 1 \Rightarrow \eta\mu\omega\sigma\upsilon\nu\omega = \frac{\lambda^2 - 1}{2}$$

β) $\eta\mu^3\omega + \sigma\upsilon\nu^3\omega = (\eta\mu\omega + \sigma\upsilon\nu\omega)(\eta\mu^2\omega - \eta\mu\omega\sigma\upsilon\nu\omega + \sigma\upsilon\nu^2\omega) =$

$$\begin{aligned} \eta\mu\omega + \sigma\upsilon\nu\omega = \lambda, \quad \eta\mu\omega\sigma\upsilon\nu\omega = \frac{\lambda^2 - 1}{2}, \quad \eta\mu^2\omega + \sigma\upsilon\nu^2\omega = 1 \\ = \lambda \left(1 - \frac{\lambda^2 - 1}{2} \right) = \lambda \frac{2 - \lambda^2 + 1}{2} = \frac{3\lambda - \lambda^3}{2} \end{aligned}$$

γ) $\eta\mu^4\omega + \sigma\upsilon\nu^4\omega = (\eta\mu^2\omega + \sigma\upsilon\nu^2\omega)^2 - 2\eta\mu^2\omega\sigma\upsilon\nu^2\omega \stackrel{\eta\mu^2\omega + \sigma\upsilon\nu^2\omega = 1}{=} \stackrel{\eta\mu\omega\sigma\upsilon\nu\omega = \frac{\lambda^2 - 1}{2}}{=} 1^2 - 2 \left(\frac{\lambda^2 - 1}{2} \right)^2 =$

$$= 1 - 2 \frac{\lambda^4 - 2\lambda^2 + 1}{4} = \frac{2 - \lambda^4 + 2\lambda^2 - 1}{2} = \frac{1 - \lambda^4 + 2\lambda^2}{2}$$

13.12 2)

α) $\eta\mu\omega + \sigma\upsilon\nu\omega = \frac{1}{2} \Rightarrow (\eta\mu\omega + \sigma\upsilon\nu\omega)^2 = \frac{1}{4} \Rightarrow$

$$\Rightarrow \eta\mu^2\omega + 2\eta\mu\omega \cdot \sigma\upsilon\nu\omega + \sigma\upsilon\nu^2\omega = \frac{1}{4} \Rightarrow 1 + 2\eta\mu\omega \cdot \sigma\upsilon\nu\omega = \frac{1}{4} \Rightarrow$$

$$\Rightarrow 2\eta\mu\omega \cdot \sigma\upsilon\nu\omega = \frac{-3}{4} \Rightarrow \eta\mu\omega \cdot \sigma\upsilon\nu\omega = \frac{-3}{8}$$

β) $\frac{1}{\eta\mu\omega} + \frac{1}{\sigma\upsilon\nu\omega} = \frac{\sigma\upsilon\nu\omega + \eta\mu\omega}{\eta\mu\omega + \sigma\upsilon\nu\omega} \stackrel{\eta\mu\omega \cdot \sigma\upsilon\nu\omega = \frac{-3}{8}}{=} \frac{\frac{1}{2}}{\frac{-3}{8}} = -\frac{4}{3}$

γ) $\varepsilon\phi\omega + \sigma\phi\omega = \frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} + \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega} = \frac{\eta\mu^2\omega + \sigma\upsilon\nu^2\omega}{\eta\mu\omega \cdot \sigma\upsilon\nu\omega} = \frac{1}{\eta\mu\omega \cdot \sigma\upsilon\nu\omega} \stackrel{\eta\mu\omega \cdot \sigma\upsilon\nu\omega = \frac{-3}{8}}{=} -\frac{8}{3}$

δ) $\eta\mu^3\omega + \sigma\upsilon\nu^3\omega = (\eta\mu\omega + \sigma\upsilon\nu\omega)(\eta\mu^2\omega + \eta\mu\omega \cdot \sigma\upsilon\nu\omega + \sigma\upsilon\nu^2\omega) =$

$$= (\eta\mu\omega + \sigma\upsilon\nu\omega) \left(1 + \eta\mu\omega \cdot \sigma\upsilon\nu\omega \right) = \frac{1}{2} \left(1 + \left(\frac{-3}{8} \right) \right) = \frac{5}{16}$$

13.12 3)

α) $\eta\mu\omega + \sigma\upsilon\nu\omega = \alpha \Rightarrow (\eta\mu\omega + \sigma\upsilon\nu\omega)^2 = \alpha^2 \Rightarrow$

$$\Rightarrow \eta\mu^2\omega + 2\eta\mu\omega \cdot \sigma\upsilon\nu\omega + \sigma\upsilon\nu^2\omega = \alpha^2 \Rightarrow 1 + 2\eta\mu\omega \cdot \sigma\upsilon\nu\omega = \alpha^2 \Rightarrow$$

$$\Rightarrow 2\eta\mu\omega \cdot \sigma\upsilon\nu\omega = \alpha^2 - 1 \Rightarrow \eta\mu\omega \cdot \sigma\upsilon\nu\omega = \frac{\alpha^2 - 1}{2}$$

β) $\varepsilon\phi\omega + \sigma\phi\omega = \frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} + \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega} = \frac{\eta\mu^2\omega + \sigma\upsilon\nu^2\omega}{\eta\mu\omega \cdot \sigma\upsilon\nu\omega} = \frac{1}{\eta\mu\omega \cdot \sigma\upsilon\nu\omega} = \frac{1}{\frac{\alpha^2 - 1}{2}} = \frac{2}{\alpha^2 - 1}$

γ) $\frac{1}{\eta\mu^2\omega} + \frac{1}{\sigma\upsilon\nu^2\omega} = \frac{\sigma\upsilon\nu^2\omega + \eta\mu^2\omega}{\eta\mu^2\omega \cdot \sigma\upsilon\nu^2\omega} = \frac{1}{\eta\mu^2\omega \cdot \sigma\upsilon\nu^2\omega} =$

$$= \frac{1}{\left(\frac{\alpha^2-1}{2}\right)^2} = \frac{1}{\frac{\alpha^2-2\alpha+1}{4}} = \frac{4}{\alpha^2-2\alpha+1}$$