Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

18.9 1)

α) $\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{\ln x^x} = \lim_{x\to 0^+} e^{x\ln x} \overset{\substack{\text{θέτουμε } y=x\ln x \\ \text{όταν } x\to 0^+,\ y\to 0}}{=} e^0 = 1$

διότι $\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \underset{\text{DLH}}{\overset{\frac{-\infty}{+\infty}}{=}} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+}(-x) = 0$

β) $\lim_{x\to 0^+} (1+3x)^{\frac{1}{3x}} = \lim_{x\to 0^+} e^{\ln(1+3x)^{\frac{1}{3x}}} = \lim_{x\to 0^+} e^{\frac{1}{3x}\ln(1+3x)} \overset{\substack{\text{θέτουμε } y=\frac{\ln(1+3x)}{3x} \\ \text{όταν } x\to 0^+,\ y\to 1}}{=} e^1 = e$

διότι

$$\lim_{x\to 0^+} \frac{1}{3x}\ln(1+3x) = \lim_{x\to 0^+} \frac{\ln(1+3x)}{3x} \underset{\text{DLH}}{\overset{\frac{0}{0}}{=}} \lim_{x\to 0^+} \frac{\frac{1}{1+3x}\cdot 3}{3} = \frac{1}{1+0} = 1$$

18.9 2)

$\lim_{x\to 0^+} (\eta\mu x)^{\eta\mu x} = \lim_{x\to 0^+} e^{\ln(\eta\mu x)^{\eta\mu x}} = \lim_{x\to 0^+} e^{\eta\mu x\ln\eta\mu x} \overset{(\text{επεξήγηση})}{=} e^0 = \boxed{1}$

επεξήγηση

$$\boxed{\lim_{x\to 0^+} \eta\mu x\ln\eta\mu x} = \lim_{x\to 0^+} \frac{\ln\eta\mu x}{\frac{1}{\eta\mu x}} \underset{\text{DLH}}{\overset{\frac{-\infty}{+\infty}}{=}} \lim_{x\to 0^+} \frac{\frac{1}{\eta\mu x}\sigma\upsilon\nu x}{-\frac{1}{\eta\mu^2 x}\sigma\upsilon\nu x} = \lim_{x\to 0^+}(-\eta\mu x) = \boxed{0}$$

18.9 3)

$\lim_{x\to 0^+} (\varepsilon\varphi x)^x = \lim_{x\to 0^+} e^{\ln(\varepsilon\varphi x)^x} = \lim_{x\to 0^+} e^{x\ln\varepsilon\varphi x} \overset{(\text{επεξήγηση})}{=} e^0 = \boxed{1}$

επεξήγηση

$$\boxed{\lim_{x\to 0^+} x\ln\varepsilon\varphi x} = \lim_{x\to 0^+} \frac{\ln\varepsilon\varphi x}{\frac{1}{x}} \underset{\text{DLH}}{\overset{\frac{-\infty}{+\infty}}{=}} \lim_{x\to 0^+} \frac{\frac{1}{\varepsilon\varphi x}\cdot\frac{1}{\sigma\upsilon\nu^2 x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{\frac{\sigma\upsilon\nu x}{\eta\mu x}\cdot\frac{1}{\sigma\upsilon\nu^2 x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{-x^2}{\eta\mu x\cdot\sigma\upsilon\nu x} =$$

$$= \lim_{x\to 0^+} \frac{x}{\eta\mu x}\cdot\frac{-x}{\sigma\upsilon\nu x} = 1\cdot\frac{0}{1} = 0$$

18.9 4)

$\lim_{x\to 0^+} (e^{2x}-1)^{2x} = \lim_{x\to 0^+} e^{\ln(e^{2x}-1)^{2x}} = \lim_{x\to 0^+} e^{2x\ln(e^{2x}-1)} \overset{(\text{επεξήγηση})}{=} e^0 = \boxed{1}$

επεξήγηση

$$\boxed{\lim_{x\to0^+} 2x\ln\left(e^{2x}-1\right)} = \lim_{x\to0^+}\frac{\ln\left(e^{2x}-1\right)}{\frac{1}{2x}} \overset{\frac{-\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to0^+}\frac{\frac{1}{e^{2x}-1}\cancel{2}e^{2x}}{-\frac{1}{4x^2}\cancel{2}} = \lim_{x\to0^+}\frac{4x^2e^{2x}}{e^{2x}-1} =$$

$$= \lim_{x\to0^+}4e^{2x}\cdot\lim_{x\to0^+}\frac{x^2}{e^{2x}-1} = 4\cdot\lim_{x\to0^+}\frac{x^2}{e^{2x}-1} \overset{\frac{0}{0}}{\underset{DLH}{=}} 4\cdot\lim_{x\to0^+}\frac{2x}{2e^{2x}} = 4\cdot0 = \boxed{0}$$

18.9 5)

$$\lim_{x\to0^+}\left(\varepsilon\varphi\frac{x}{2}\right)^{\frac{1}{\ln x}} = \lim_{x\to0^+} e^{\ln\left(\varepsilon\varphi\frac{x}{2}\right)^{\frac{1}{\ln x}}} = \lim_{x\to0^+} e^{\frac{1}{\ln x}\ln\left(\varepsilon\varphi\frac{x}{2}\right)} \overset{(\text{επεξήγηση})}{=} e^1 = \boxed{e}$$

επεξήγηση

$$\boxed{\lim_{x\to0^+}\frac{1}{\ln x}\ln\left(\varepsilon\varphi\frac{x}{2}\right)} = \lim_{x\to0^+}\frac{\ln\left(\varepsilon\varphi\frac{x}{2}\right)}{\ln x} \overset{\frac{-\infty}{-\infty}}{\underset{DLH}{=}} \lim_{x\to0^+}\frac{\frac{1}{\varepsilon\varphi\frac{x}{2}}\cdot\frac{1}{\sigma\upsilon\nu^2\frac{x}{2}}\cdot\frac{1}{2}}{\frac{1}{x}} =$$

$$= \lim_{x\to0^+}\frac{x}{2\cdot\varepsilon\varphi\frac{x}{2}\cdot\sigma\upsilon\nu^2\frac{x}{2}} = \lim_{x\to0^+}\frac{1}{2\cdot\sigma\upsilon\nu^2\frac{x}{2}}\cdot\lim_{x\to0^+}\frac{x}{\varepsilon\varphi\frac{x}{2}} = \frac{1}{2\cdot\sigma\upsilon\nu0}\cdot\lim_{x\to0^+}\frac{x}{\varepsilon\varphi\frac{x}{2}} \overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$= \frac{1}{2}\cdot\lim_{x\to0^+}\frac{1}{\frac{1}{\sigma\upsilon\nu^2\frac{x}{2}}\cdot\frac{1}{2}} = \frac{1}{2}\cdot\lim_{x\to0^+}\frac{1}{\frac{1}{2\sigma\upsilon\nu^2 0}} = \frac{1}{2}\cdot2 = \boxed{1}$$

18.9 6)

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to+\infty} e^{\ln\left(1+\frac{1}{x}\right)^x} = \lim_{x\to+\infty} e^{x\ln\left(1+\frac{1}{x}\right)} \overset{(\text{επεξήγηση})}{=} e^1 = \boxed{e}$$

επεξήγηση

$$\boxed{\lim_{x\to+\infty} x\ln\left(1+\frac{1}{x}\right)} = \lim_{x\to+\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to+\infty}\frac{\frac{1}{1+\frac{1}{x}}\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \lim_{x\to+\infty}\frac{1}{\frac{x+1}{x}} =$$

$$= \lim_{x\to+\infty}\frac{x}{x+1} \overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}} \lim_{x\to+\infty}\frac{1}{1} = \boxed{1}$$

18.9 7)

$$\lim_{x\to1} x^{\frac{2}{x-1}} = \lim_{x\to1} e^{\ln x^{\frac{2}{x-1}}} = \lim_{x\to1} e^{\frac{2}{x-1}\ln x} \overset{(\text{επεξήγηση})}{=} \boxed{e^2}$$

επεξήγηση

$$\boxed{\lim_{x\to1}\frac{2}{x-1}\ln x} = \lim_{x\to1}\frac{2\ln x}{x-1} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x\to1}\frac{\frac{2}{x}}{1} = \boxed{2}$$

18.9 8)

$$\lim_{x\to2^+}(x-2)^{\ln(x-1)}=\lim_{x\to2^+}e^{\ln(x-2)^{\ln(x-1)}}=\lim_{x\to2^+}e^{\ln(x-1)\cdot\ln(x-2)}\overset{(\text{επεξήγηση})}{=}e^0=\boxed{1}$$

επεξήγηση

$$\boxed{\lim_{x\to2^+}\ln(x-1)\cdot\ln(x-2)}=\lim_{x\to2^+}\frac{\ln(x-2)}{\frac{1}{\ln(x-1)}}\underset{DLH}{\overset{\frac{-\infty}{+\infty}}{=}}\lim_{x\to2^+}\frac{\frac{1}{x-2}}{-\frac{1}{\ln^2(x-1)}\cdot\frac{1}{x-1}}=$$

$$=\lim_{x\to2^+}\frac{-(x-1)\ln^2(x-1)}{x-2}=\lim_{x\to2^+}\left[-(x-1)\right]\cdot\lim_{x\to2^+}\frac{\ln^2(x-1)}{x-2}=-1\cdot\lim_{x\to2^+}\frac{\ln^2(x-1)}{x-2}\underset{DLH}{\overset{\frac{0}{0}}{=}}$$

$$=-1\cdot\lim_{x\to2^+}\frac{2\ln(x-1)\cdot\frac{1}{x-1}}{1}=-1\cdot2\cdot0\cdot\frac{1}{1}=\boxed{0}$$

18.9 9)

$$\lim_{x\to0^+}3x^{\frac{1}{\ln(e^{3x}-1)}}=3\cdot\lim_{x\to0^+}e^{\ln x^{\frac{1}{\ln(e^{3x}-1)}}}=3\cdot\lim_{x\to0^+}e^{\frac{1}{\ln(e^{3x}-1)}\ln x}\overset{(\text{επεξήγηση})}{=}3\cdot e^1=\boxed{3e}$$

επεξήγηση

$$\boxed{\lim_{x\to0^+}\frac{1}{\ln(e^{3x}-1)}\ln x}=\lim_{x\to0^+}\frac{\ln x}{\ln(e^{3x}-1)}\underset{DLH}{\overset{\frac{-\infty}{-\infty}}{=}}\lim_{x\to0^+}\frac{\frac{1}{x}}{\frac{1}{e^{3x}-1}\cdot3e^{3x}}=\lim_{x\to0^+}\frac{(e^{3x}-1)}{3xe^{3x}}=$$

$$=\lim_{x\to0^+}\frac{1}{3e^{3x}}\cdot\lim_{x\to0^+}\frac{e^{3x}-1}{x}=\frac{1}{3}\cdot\lim_{x\to0^+}\frac{e^{3x}-1}{x}\underset{DLH}{\overset{\frac{0}{0}}{=}}\frac{1}{3}\cdot\lim_{x\to0^+}\frac{3e^{3x}}{1}=\frac{1}{3}\cdot3=\boxed{1}$$

18.9 10)

$$\lim_{x\to+\infty}\left(1+\frac{3}{x}\right)^{x^3}=\lim_{x\to+\infty}e^{\ln\left(1+\frac{3}{x}\right)^{x^3}}=\lim_{x\to+\infty}e^{x^3\ln\left(1+\frac{3}{x}\right)}\overset{(\text{επεξήγηση})}{=}e^{+\infty}=\boxed{+\infty}$$

επεξήγηση

$$\boxed{\lim_{x\to+\infty}x^3\ln\left(1+\frac{3}{x}\right)}=\lim_{x\to+\infty}\frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{x^3}}\underset{DLH}{\overset{\frac{0}{0}}{=}}\lim_{x\to+\infty}\frac{\frac{1}{1+\frac{3}{x}}\cdot\left(-\frac{\cancel{3}}{x^2}\right)}{-\frac{1}{x^{\cancel{6}4}}\cdot\cancel{3}\cancel{x^2}}=\lim_{x\to+\infty}\frac{\frac{1}{\left(1+\frac{3}{x}\right)\cancel{x^2}}}{\frac{1}{x^{\cancel{4}2}}}=$$

$$=\lim_{x\to+\infty}\frac{x^2}{1+\frac{3}{x}}=\frac{+\infty}{1+0}=\boxed{+\infty}$$

18.9 11)

$$\lim_{x\to0^+}(1+3x)^{\frac{1}{3x}}=\lim_{x\to0^+}e^{\ln(1+3x)^{\frac{1}{3x}}}=\lim_{x\to0^+}e^{\frac{1}{3x}\ln(1+3x)}\overset{(\text{επεξήγηση})}{=}e^1=\boxed{e}$$

επεξήγηση

$$\boxed{\lim_{x\to0^+}\frac{1}{3x}\ln(1+3x)}=\lim_{x\to0^+}\frac{\ln(1+3x)}{3x}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0^+}\frac{\frac{1}{1+3x}\cdot\cancel{3}}{\cancel{3}}=\frac{1}{1+0}=\boxed{1}$$

18.9 12)

$$\lim_{x\to+\infty}\left(1-\frac{1}{x}\right)^x=\lim_{x\to+\infty}e^{\ln\left(1-\frac{1}{x}\right)^x}=\lim_{x\to+\infty}e^{x\ln\left(1-\frac{1}{x}\right)}\overset{(\text{επεξήγηση})}{=}e^{-1}=\boxed{\frac{1}{e}}$$

επεξήγηση

$$\boxed{\lim_{x\to+\infty}x\ln\left(1-\frac{1}{x}\right)}=\lim_{x\to+\infty}\frac{\ln\left(1-\frac{1}{x}\right)}{\frac{1}{x}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{\frac{1}{1-\frac{1}{x}}\cdot\cancel{\frac{1}{x^2}}}{-\cancel{\frac{1}{x^2}}}=-\frac{1}{1-0}=\boxed{-1}$$

18.9 13)

$$\lim_{x\to0}(\eta\mu3x+\sigma\upsilon\nu2x)^{\frac{3}{x}}=\lim_{x\to0}e^{\ln(\eta\mu3x+\sigma\upsilon\nu2x)^{\frac{3}{x}}}=\lim_{x\to0}e^{\frac{3}{x}\cdot\ln(\eta\mu3x+\sigma\upsilon\nu2x)}\overset{(\text{επεξήγηση})}{=}\boxed{e^9}$$

επεξήγηση

$$\boxed{\lim_{x\to0}\frac{3}{x}\cdot\ln(\eta\mu3x+\sigma\upsilon\nu2x)}=\lim_{x\to0}\frac{3\ln(\eta\mu3x+\sigma\upsilon\nu2x)}{x}\overset{\frac{0}{0}}{\underset{DLH}{=}}$$

$$=\lim_{x\to0}\frac{\frac{3}{\eta\mu3x+\sigma\upsilon\nu2x}\cdot(3\sigma\upsilon\nu3x-2\eta\mu2x)}{1}=\frac{3(3\sigma\upsilon\nu0-2\eta\mu0)}{\eta\mu0+\sigma\upsilon\nu0}=\boxed{9}$$