

6.B1

Είναι $|\vec{\alpha}| = |\vec{\beta}| = 1$ και $\vec{\beta}^2 = -2\vec{\alpha} \cdot \vec{\beta} = |\vec{\beta}|^2$

Έχουμε $\cos(\vec{\alpha}, \vec{\beta}) = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}| \cdot |\vec{\beta}|} = \frac{\frac{|\vec{\beta}|^2}{-2}}{1 \cdot 1} = \frac{-1}{2}$

Οπότε $\cos(\vec{\alpha}, \vec{\beta}) = -\frac{1}{2} \Rightarrow \left(\vec{\alpha}, \vec{\beta}\right) = \frac{2\pi}{3}$

6.B2

α) $\vec{AB} = (8-2, -1-1) = (6, -2)$

$\vec{BG} = (4-8, -3-(-1)) = (-4, -2)$

$\vec{GL} = (1-4, -2-(-3)) = (-3, 1)$

$\vec{AL} = (1-2, -2-1) = (-1, -3)$

β) $\vec{AB} = (6, -2) = -2(-3, 1) = -2\vec{GL} \Rightarrow \vec{AB} \parallel \vec{GL}$

γ) $\vec{AL} \cdot \vec{AB} = (-1, -3) \cdot (6, -2) = -6 - 3(-2) = -6 + 6 = 0$

Άρα $\vec{AL} \cdot \vec{AB} = 0 \Rightarrow \vec{AL} \perp \vec{AB}$

δ) $\lambda_{\vec{BG}} = \frac{-2}{-4} = \frac{1}{2}$

6.B3

Είναι $2(\vec{x} - 3\vec{\beta}) = 3(4\vec{x} + \vec{\alpha} - 2\vec{\beta}) \Rightarrow$

$\Rightarrow 2\vec{x} - 6\vec{\beta} = 12\vec{x} + 3\vec{\alpha} - 6\vec{\beta} \Rightarrow$

$\Rightarrow 2\vec{x} = 12\vec{x} + 3\vec{\alpha} \Rightarrow 10\vec{x} = -3\vec{\alpha} \Rightarrow \vec{x} = -\frac{3}{10}\vec{\alpha}$

Άρα $\vec{x} = \lambda\vec{\alpha}$ με $\lambda = -\frac{3}{10} < 0$ συνεπώς $\vec{x} \uparrow \downarrow \vec{\alpha}$

6.B4

Είναι $\vec{AB} = (-2-(-4), -1-3) = (2, -4)$

και $\vec{AG} = (\lambda+1-(-4), -\lambda-9-3) = (\lambda+5, -\lambda-12)$

Για να είναι τα Α, Β, Γ συνευθειακά, πρέπει:

$\det(\vec{AB}, \vec{AG}) = 0 \Rightarrow \begin{vmatrix} 2 & -4 \\ \lambda+5 & -\lambda-12 \end{vmatrix} = 0 \Rightarrow -2\lambda - 24 + 4\lambda + 20 = 0 \Rightarrow$

$\Rightarrow 2\lambda - 4 = 0 \Rightarrow \lambda = 2$

6.B5

α) $\vec{\alpha} \cdot \vec{\beta} = |\vec{\alpha}| \cdot |\vec{\beta}| \cdot \cos(\vec{\alpha}, \vec{\beta}) = 2 \cdot 4 \cdot \cos\frac{\pi}{3} = 2 \cdot 4 \cdot \frac{1}{2} = 4$

β) $\vec{\alpha} \cdot \vec{\gamma} = \vec{\alpha} \cdot (\vec{\alpha} - \vec{\beta}) = \vec{\alpha}^2 - \vec{\alpha} \cdot \vec{\beta} = |\vec{\alpha}|^2 - \vec{\alpha} \cdot \vec{\beta} = 2^2 - 4 = 0$

γ) $\vec{\alpha} \cdot \vec{\gamma} = 0 \Rightarrow \vec{\alpha} \perp \vec{\gamma} \Rightarrow \left(\vec{\alpha}, \vec{\gamma}\right) = \frac{\pi}{2}$

6.B6

$$\alpha) \quad \vec{\alpha} \cdot (2\vec{\alpha} - \vec{\beta}) = 2\vec{\alpha}^2 - \vec{\alpha} \cdot \vec{\beta} = 2(2,1) \cdot (2,1) - (2,1) \cdot (-3,2) = \\ = 2(2^2 + 1^2) - (2(-3) + 2) = 2 \cdot 5 + 4 = 14$$

$$\beta) \quad \vec{\gamma} \perp \vec{\alpha} \Rightarrow \vec{\gamma} \cdot \vec{\alpha} = 0 \Rightarrow (x, y) \cdot (2, 1) = 0 \Rightarrow 2x + y = 0 \quad (1)$$

$$\text{και} \quad |\vec{\gamma}| = \sqrt{5} \Rightarrow \sqrt{x^2 + y^2} = \sqrt{5} \Rightarrow x^2 + y^2 = 5 \quad (2)$$

Λύνουμε το σύστημα (1), (2):

$$\begin{cases} 2x + y = 0 \\ x^2 + y^2 = 5 \end{cases} \Rightarrow \begin{cases} y = -2x \\ x^2 + (-2x)^2 = 5 \end{cases} \Rightarrow \begin{cases} y = -2x \\ 5x^2 = 5 \end{cases} \Rightarrow \begin{cases} y = -2 & \text{ή} & y = 2 \\ x = 1 & \text{ή} & x = -1 \end{cases}$$

$$\text{Άρα} \quad \vec{\gamma} = (1, -2) \quad \text{ή} \quad \vec{\gamma} = (-1, 2)$$