

$$\begin{aligned}
 \text{Έχουμε : } |\overrightarrow{ΑΓ}|^2 \cdot |\overrightarrow{ΔΒ}|^2 &= |\overrightarrow{ΑΔ} + \overrightarrow{ΔΓ}|^2 \cdot |\overrightarrow{ΔΑ} + \overrightarrow{ΑΒ}|^2 = \\
 &= \left(|\overrightarrow{ΑΔ}|^2 + 2|\overrightarrow{ΑΔ}| \cdot |\overrightarrow{ΔΓ}| \cdot \cos \frac{\pi}{4} + |\overrightarrow{ΔΓ}|^2 \right) \cdot \left(|\overrightarrow{ΔΑ}|^2 + 2|\overrightarrow{ΔΑ}| \cdot |\overrightarrow{ΑΒ}| \cdot \cos \left(\pi - \frac{\pi}{4} \right) + |\overrightarrow{ΑΒ}|^2 \right) = \\
 &= \left(|\vec{\beta}|^2 + 2|\vec{\alpha}| \cdot |\vec{\beta}| \cdot \frac{\sqrt{2}}{2} + |\vec{\alpha}|^2 \right) \cdot \left(|\vec{\beta}|^2 + 2|\vec{\alpha}| \cdot |\vec{\beta}| \cdot \left(-\frac{\sqrt{2}}{2} \right) + |\vec{\alpha}|^2 \right) = \\
 &= \left(|\vec{\beta}|^2 + \sqrt{2}|\vec{\alpha}| \cdot |\vec{\beta}| + |\vec{\alpha}|^2 \right) \left(|\vec{\beta}|^2 - \sqrt{2}|\vec{\alpha}| \cdot |\vec{\beta}| + |\vec{\alpha}|^2 \right) = \\
 &= \left[\left(|\vec{\beta}|^2 + |\vec{\alpha}|^2 \right) + \sqrt{2}|\vec{\alpha}||\vec{\beta}| \right] \left[\left(|\vec{\beta}|^2 + |\vec{\alpha}|^2 \right) - \sqrt{2}|\vec{\alpha}||\vec{\beta}| \right] = \\
 &\stackrel{\text{διαφορά τετραγώνων}}{=} \left(|\vec{\beta}|^2 + |\vec{\alpha}|^2 \right)^2 - \sqrt{2}^2 |\vec{\alpha}|^2 |\vec{\beta}|^2 = \\
 &= |\vec{\beta}|^4 + 2|\vec{\alpha}|^2 \cdot |\vec{\beta}|^2 + |\vec{\alpha}|^4 - 2|\vec{\alpha}|^2 \cdot |\vec{\beta}|^2 = |\vec{\beta}|^4 + |\vec{\alpha}|^4
 \end{aligned}$$