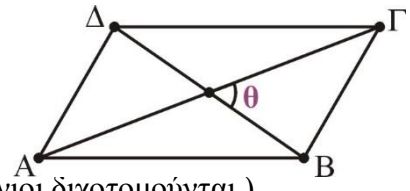


5.45

Έστω O το σημείο τομής των διαγωνίων AG , BD

Τότε είναι $\hat{\theta} = \left(\overrightarrow{OA}, \overrightarrow{OD} \right)$



Παρατηρούμε ότι : $\overrightarrow{OA} = \frac{1}{2} \cdot \overrightarrow{GA} = \frac{1}{2} \cdot (\overrightarrow{GD} + \overrightarrow{DA})$ (γιατί οι διαγώνιοι διχοτομούνται)

Όμοια : $\overrightarrow{OD} = \frac{1}{2} \cdot \overrightarrow{BD} = \frac{1}{2} (\overrightarrow{BA} + \overrightarrow{AD})$

Οπότε είναι : $\overrightarrow{OA} \cdot \overrightarrow{OD} = \frac{1}{2} (\overrightarrow{GD} + \overrightarrow{DA}) \cdot \frac{1}{2} (\overrightarrow{BA} + \overrightarrow{AD}) =$

$$= \frac{1}{4} [\overrightarrow{GD} \cdot \overrightarrow{BA} + \overrightarrow{GD} \cdot \overrightarrow{AD} + \overrightarrow{DA} \cdot \overrightarrow{BA} + \overrightarrow{DA} \cdot \overrightarrow{AD}] =$$

$$= \frac{1}{4} (|\overrightarrow{GD}| \cdot |\overrightarrow{BA}| \cdot \cos(\overrightarrow{GD}, \overrightarrow{BA}) + |\overrightarrow{GD}| \cdot |\overrightarrow{AD}| \cdot \cos(\overrightarrow{GD}, \overrightarrow{AD}) +$$

$$+ |\overrightarrow{DA}| \cdot |\overrightarrow{BA}| \cdot \cos(\overrightarrow{DA}, \overrightarrow{BA}) + |\overrightarrow{DA}| \cdot |\overrightarrow{AD}| \cdot \cos(\overrightarrow{DA}, \overrightarrow{AD})) =$$

$$= \frac{1}{4} \left(\sqrt{3} \cdot \sqrt{3} \cdot 1 + \sqrt{3} \cdot 1 \cdot \cos\left(\pi - \frac{\pi}{6}\right) + 1 \cdot \sqrt{3} \cdot \cos\frac{\pi}{6} + 1 \cdot 1 \cdot (-1) \right) =$$

$$= \frac{1}{4} \left(3 + \sqrt{3} \cdot \left(-\frac{\sqrt{3}}{2}\right) + \sqrt{3} \cdot \frac{\sqrt{3}}{2} - 1 \right) = \frac{1}{2}$$

$$\text{Ακόμη : } |\overrightarrow{OA}|^2 = \frac{1}{4} |\overrightarrow{GD} + \overrightarrow{DA}|^2 = \frac{1}{4} (|\overrightarrow{GD}|^2 + 2|\overrightarrow{GD}| \cdot |\overrightarrow{DA}| \cos(\overrightarrow{GD}, \overrightarrow{DA}) + |\overrightarrow{DA}|^2) =$$

$$= \frac{1}{4} \left(\sqrt{3}^2 + 2 \cdot \sqrt{3} \cdot 1 \cdot \cos\frac{\pi}{6} + 1^2 \right) =$$

$$= \frac{1}{4} (3 + 3 + 1) = \frac{7}{4} \Rightarrow |\overrightarrow{OA}| = \frac{\sqrt{7}}{2}$$

$$\text{Όμοια : } |\overrightarrow{OD}|^2 = \frac{1}{4} |\overrightarrow{BA} + \overrightarrow{AD}|^2 = \frac{1}{4} \left(\sqrt{3}^2 - 2 \cdot \sqrt{3} \cdot \frac{1 \cdot \sqrt{3}}{2} + 1^2 \right) = \frac{1}{4} \Rightarrow |\overrightarrow{OD}| = \frac{1}{2}$$

$$\text{Οπότε : } \cos \hat{\theta} = \frac{\overrightarrow{OA} \cdot \overrightarrow{OD}}{|\overrightarrow{OA}| \cdot |\overrightarrow{OD}|} = \frac{\frac{1}{2}}{\frac{\sqrt{7}}{2} \cdot \frac{1}{2}} = \frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$