

5.29 1)

$$\begin{aligned} \text{Έχουμε } \vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0} &\Rightarrow (\vec{\alpha} + \vec{\beta} + \vec{\gamma})^2 = \vec{0}^2 \Rightarrow \vec{\alpha}^2 + \vec{\beta}^2 + \vec{\gamma}^2 + 2\vec{\alpha} \cdot \vec{\beta} + 2\vec{\alpha} \cdot \vec{\gamma} + 2\vec{\beta} \cdot \vec{\gamma} = 0 \Rightarrow \\ &\Rightarrow |\vec{\alpha}|^2 + |\vec{\beta}|^2 + |\vec{\gamma}|^2 + 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma}) = 0 \Rightarrow 1^2 + 2^2 + 3^2 + 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma}) = 0 \Rightarrow \\ &\Rightarrow 14 + 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma}) = 0 \Rightarrow 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma}) = -14 \Rightarrow \vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma} = -7 \end{aligned}$$

5.29 2)

$$\begin{aligned} \text{Έχουμε } \vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0} &\Rightarrow (\vec{\alpha} + \vec{\beta} + \vec{\gamma})^2 = \vec{0}^2 \Rightarrow \\ &\Rightarrow \vec{\alpha}^2 + \vec{\beta}^2 + \vec{\gamma}^2 + 2\vec{\alpha} \cdot \vec{\beta} + 2\vec{\alpha} \cdot \vec{\gamma} + 2\vec{\beta} \cdot \vec{\gamma} = 0 \Rightarrow \\ &\Rightarrow |\vec{\alpha}|^2 + |\vec{\beta}|^2 + |\vec{\gamma}|^2 + 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma}) = 0 \Rightarrow \\ &\Rightarrow 3^2 + 4^2 + 5^2 + 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma}) = 0 \Rightarrow \\ &\Rightarrow 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma}) = -50 \Rightarrow \vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma} = -25 \end{aligned}$$

5.29 3)

$$\begin{aligned} \text{Έχουμε } \vec{\alpha} = \vec{\beta} + \vec{\gamma} &\Rightarrow \vec{\alpha} - \vec{\beta} - \vec{\gamma} = \vec{0} \Rightarrow (\vec{\alpha} - \vec{\beta} - \vec{\gamma})^2 = \vec{0}^2 \Rightarrow \\ &\Rightarrow \vec{\alpha}^2 + \vec{\beta}^2 + \vec{\gamma}^2 - 2\vec{\alpha} \cdot \vec{\beta} + 2\vec{\beta} \cdot \vec{\gamma} - 2\vec{\alpha} \cdot \vec{\gamma} = 0 \Rightarrow \\ &\Rightarrow |\vec{\alpha}|^2 + |\vec{\beta}|^2 + |\vec{\gamma}|^2 - 2\vec{\alpha} \cdot \vec{\beta} + 2\vec{\beta} \cdot \vec{\gamma} - 2\vec{\alpha} \cdot \vec{\gamma} = 0 \Rightarrow \\ &\Rightarrow 1^2 + 1^2 + 1^2 - 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} - \vec{\beta} \cdot \vec{\gamma}) = 0 \Rightarrow \\ &\Rightarrow \vec{\alpha} \cdot \vec{\beta} + \vec{\beta} \cdot \vec{\gamma} - \vec{\alpha} \cdot \vec{\gamma} = \frac{3}{2} \end{aligned}$$

5.29 4)

$$\begin{aligned} \text{Έχουμε } \vec{BG} + \vec{GA} + \vec{AB} &= \vec{0} \Rightarrow \\ &\Rightarrow \vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0} \Rightarrow (\vec{\alpha} + \vec{\beta} + \vec{\gamma})^2 = \vec{0}^2 \Rightarrow \\ &\Rightarrow \vec{\alpha}^2 + \vec{\beta}^2 + \vec{\gamma}^2 + 2\vec{\alpha} \cdot \vec{\beta} + 2\vec{\alpha} \cdot \vec{\gamma} + 2\vec{\beta} \cdot \vec{\gamma} = 0 \Rightarrow \\ &\Rightarrow |\vec{\alpha}|^2 + |\vec{\beta}|^2 + |\vec{\gamma}|^2 + 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma}) = 0 \Rightarrow \\ &\Rightarrow 2^2 + 2^2 + 2^2 + 2(\vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma}) = 0 \Rightarrow \\ &\Rightarrow \vec{\alpha} \cdot \vec{\beta} + \vec{\alpha} \cdot \vec{\gamma} + \vec{\beta} \cdot \vec{\gamma} = -6 \end{aligned}$$