Β ΛΥΚΕΙΟΥ ΘΕΤΙΚΟΣ ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΣ

5.27 1)

Έχουμε

$$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB} \Rightarrow \overrightarrow{AB}^2 = \left(\overrightarrow{AM} + \overrightarrow{MB}\right)^2 \Rightarrow$$

$$\Rightarrow \overrightarrow{AB}^2 = \overrightarrow{AM}^2 + 2\overrightarrow{AM}\cdot\overrightarrow{MB} + \overrightarrow{MB}^2 \quad (1)$$

Επίσης

$$\overrightarrow{A\Gamma} = \overrightarrow{AM} + \overrightarrow{M\Gamma} \Rightarrow \overrightarrow{A\Gamma}^2 = \left(\overrightarrow{AM} + \overrightarrow{M\Gamma}\right)^2 \Rightarrow \overrightarrow{A\Gamma}^2 = \overrightarrow{AM}^2 + 2\overrightarrow{AM}\cdot\overrightarrow{MB} + \overrightarrow{MB}^2 \quad (2)$$

Προσθέτουμε τις (1) και (2) κατά μέλη και έχουμε

$$\overrightarrow{AB}^2 + \overrightarrow{A\Gamma}^2 = 2\overrightarrow{AM}^2 + 2\overrightarrow{MB}^2 + 2\overrightarrow{AM}\cdot\overrightarrow{MB} + 2\overrightarrow{AM}\cdot\overrightarrow{MB} \Rightarrow$$

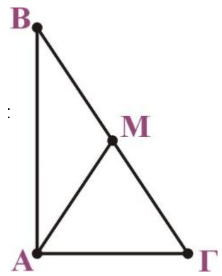
$$\Rightarrow \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{A\Gamma}\right|^2 = 2\left|\overrightarrow{AM}\right|^2 + 2\left|\overrightarrow{MB}\right|^2 + 2\overrightarrow{AM}\cdot\left(\overrightarrow{MB} + \overrightarrow{MB}\right) \Rightarrow$$

$$\Rightarrow \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{A\Gamma}\right|^2 = 2\left|\overrightarrow{AM}\right|^2 + 2\left(\frac{\left|\overrightarrow{B\Gamma}\right|}{2}\right)^2 + 2\overrightarrow{AM}\cdot\vec{0} \Rightarrow \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{A\Gamma}\right|^2 = 2\left|\overrightarrow{AM}\right|^2 + 2\frac{\left|\overrightarrow{B\Gamma}\right|^2}{4} \Rightarrow$$

$$\Rightarrow \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{A\Gamma}\right|^2 = 2\left|\overrightarrow{AM}\right|^2 + \frac{\left|\overrightarrow{B\Gamma}\right|^2}{2} \Rightarrow AB^2 + A\Gamma^2 = 2AM^2 + \frac{B\Gamma^2}{2}$$

5.27 2)

Έστω $\hat{A} = 90^o$ και AM η διάμεσος που αντιστοιχεί στην υποτείνουσα.



Το $A\hat{B}\Gamma$ είναι ορθογώνιο, οπότε έχουμε:

$$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{A\Gamma}\right|^2 = \left|\overrightarrow{B\Gamma}\right|^2 \Rightarrow \overrightarrow{AB}^2 + \overrightarrow{A\Gamma}^2 = \overrightarrow{B\Gamma}^2 \Rightarrow \left(\overrightarrow{AM} + \overrightarrow{MB}\right)^2 + \left(\overrightarrow{AM} + \overrightarrow{M\Gamma}\right)^2 = \overrightarrow{B\Gamma}^2$$

$$\overset{\overrightarrow{M\Gamma} = -\overrightarrow{MB}}{\Rightarrow} \left(\overrightarrow{AM} + \overrightarrow{MB}\right)^2 + \left(\overrightarrow{AM} - \overrightarrow{MB}\right)^2 = \overrightarrow{B\Gamma}^2 \Rightarrow$$

$$\Rightarrow \overrightarrow{AM}^2 + 2\overrightarrow{AM}\cdot\overrightarrow{MB} + \overrightarrow{MB}^2 + \overrightarrow{AM}^2 - 2\overrightarrow{AM}\cdot\overrightarrow{MB} + \overrightarrow{MB}^2 = \overrightarrow{B\Gamma}^2 \Rightarrow$$

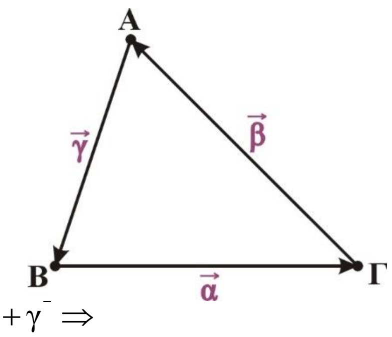
$$\Rightarrow 2\overrightarrow{AM}^2 + 2\overrightarrow{MB}^2 = \overrightarrow{B\Gamma}^2 \Rightarrow 2\overrightarrow{AM}^2 + 2\cdot\left(\frac{\overrightarrow{B\Gamma}}{2}\right)^2 = \overrightarrow{B\Gamma}^2 \Rightarrow$$

$$\Rightarrow 2\overrightarrow{AM}^2 = \frac{\overrightarrow{B\Gamma}^2}{2} \Rightarrow \overrightarrow{AM}^2 = \frac{\overrightarrow{B\Gamma}^2}{4} \Rightarrow \left|\overrightarrow{AM}\right| = \frac{\left|\overrightarrow{B\Gamma}\right|}{2}$$

5.27 3)

Έστω $\overrightarrow{B\Gamma} = \vec{\alpha}$, $\overrightarrow{\Gamma A} = \vec{\beta}$, $\overrightarrow{AB} = \vec{\gamma}$

Είναι $\vec{\alpha} = \overrightarrow{BA} + \overrightarrow{A\Gamma} = -\vec{\beta} - \vec{\gamma}$



Άρα

$$\vec{\alpha}^2 = \left(-\vec{\beta} - \vec{\gamma}\right)^2 = \vec{\beta}^2 + 2\vec{\beta}\cdot\vec{\gamma} + \vec{\gamma}^2 \Rightarrow \vec{\alpha}^2 = \vec{\beta}^2 + 2\left|\vec{\beta}\right|\cdot\left|\vec{\gamma}\right|\cdot\sigma\upsilon\nu\left(\vec{\beta},\vec{\gamma}\right) + \vec{\gamma}^2 \Rightarrow$$

$$\Rightarrow \vec{\alpha}^2 = \vec{\beta}^2 + 2\left|\vec{\beta}\right|\cdot\left|\vec{\gamma}\right|\cdot\sigma\upsilon\nu\left(180^o - \hat{A}\right) + \vec{\gamma}^2 \Rightarrow \vec{\alpha}^2 = \vec{\beta}^2 + 2\left|\vec{\beta}\right|\cdot\left|\vec{\gamma}\right|\cdot\left(-\sigma\upsilon\nu\hat{A}\right) + \vec{\gamma}^{-} \Rightarrow$$

$$\Rightarrow \vec{\alpha}^2 = \vec{\beta}^2 - 2\beta\cdot\gamma\cdot\sigma\upsilon\nu A + \vec{\gamma}^2$$