5.24 1)

Είναι $\vec{\gamma}=-3\vec{\alpha}+8\vec{\beta}\Rightarrow\vec{\gamma}=-3(2,-3)+8(1,-1)\Rightarrow\vec{\gamma}=(-6,9)+(8,-8)\Rightarrow\vec{\gamma}=(2,1)$

και $\vec{\delta}=-4\vec{\alpha}+9\vec{\beta}\Rightarrow\vec{\delta}=-4(2,-3)+9(1,-1)\Rightarrow\vec{\delta}=(-8,12)+(9,-9)\Rightarrow\vec{\delta}=(1,3)$

Οπότε $\text{συν}\left(\hat{\vec{\gamma},\vec{\delta}}\right)=\dfrac{x_1x_2+y_1y_2}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}}=\dfrac{2\cdot1+1\cdot3}{\sqrt{2^2+1^2}\cdot\sqrt{1^2+3^2}}=\dfrac{5}{\sqrt{5}\cdot\sqrt{10}}=\dfrac{5}{\sqrt{50}}=\dfrac{5}{5\sqrt{2}}=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$

Άρα $\text{συν}\left(\hat{\vec{\gamma},\vec{\delta}}\right)=\dfrac{\sqrt{2}}{2}\overset{0^o\le\hat{\vec{\gamma},\vec{\delta}}\le180^o}{\Rightarrow}(\hat{\vec{\gamma},\vec{\delta}})=45^o$

5.24 2)

Είναι $\vec{\gamma}=-11\vec{\alpha}-8\vec{\beta}=-11(1,-2)-8(-1,3)=(-11,22)-(-8,24)=(-3,-2)$

και $\vec{\delta}=8\vec{\alpha}+7\vec{\beta}=8(1,-2)+7(-1,3)=(8,-16)+(-7,21)=(1,5)$

Οπότε $\text{συν}\left(\hat{\vec{\gamma},\vec{\delta}}\right)=\dfrac{\vec{\gamma}\cdot\vec{\delta}}{|\vec{\gamma}|\cdot|\vec{\delta}|}=\dfrac{-3\cdot1-2\cdot5}{\sqrt{(-3)^2+(-2)^2}\cdot\sqrt{1^2+5^2}}=\dfrac{-13}{\sqrt{13}\cdot\sqrt{26}}=-\dfrac{13}{\sqrt{2}\cdot13}=-\dfrac{\sqrt{2}}{2}$

Επομένως $\text{συν}\left(\hat{\vec{\gamma},\vec{\delta}}\right)=-\dfrac{\sqrt{2}}{2}\Rightarrow\left(\hat{\vec{\gamma},\vec{\delta}}\right)=135^o$

5.24 3)

Είναι $\vec{\gamma}=5\vec{\alpha}-9\vec{\beta}=5\left(2\sqrt{3},3\right)-9\left(\sqrt{3},2\right)=\left(10\sqrt{3},15\right)-\left(9\sqrt{3},18\right)=\left(\sqrt{3},-3\right)$

και $\vec{\delta}=-6\vec{\alpha}+10\vec{\beta}=-6\left(2\sqrt{3},3\right)+10\left(\sqrt{3},2\right)=\left(-12\sqrt{3},-18\right)+\left(10\sqrt{3},20\right)=\left(-2\sqrt{3},2\right)$

Οπότε $\text{συν}\left(\hat{\vec{\gamma},\vec{\delta}}\right)=\dfrac{\vec{\gamma}\cdot\vec{\delta}}{|\vec{\gamma}|\cdot|\vec{\delta}|}=\dfrac{\sqrt{3}\left(-2\sqrt{3}\right)-3\cdot2}{\sqrt{\left(\sqrt{3}\right)^2+(-3)^2}\cdot\sqrt{\left(-2\sqrt{3}\right)^2+2^2}}=\dfrac{-12}{\sqrt{12}\cdot\sqrt{16}}=-\dfrac{13}{2\sqrt{3}\cdot4}=-\dfrac{\sqrt{3}}{3}$

Επομένως $\text{συν}\left(\hat{\vec{\gamma},\vec{\delta}}\right)=-\dfrac{\sqrt{3}}{3}\Rightarrow\left(\hat{\vec{\gamma},\vec{\delta}}\right)=150^o$