

Β ΛΥΚΕΙΟΥ ΘΕΤΙΚΟΣ ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΣ

5.23 1)

$$\alpha) \vec{\alpha} \cdot \vec{\beta} = |\vec{\alpha}| \cdot |\vec{\beta}| \cdot \cos(\vec{\alpha}, \vec{\beta}) = 2 \cdot 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 2 \cdot \sqrt{3}^2 = 6$$

$$\beta) \vec{\gamma} \cdot \vec{\delta} = (-2\vec{\alpha} + \vec{\beta})(\vec{\alpha} - \vec{\beta}) = -2\vec{\alpha}^2 + 2\vec{\alpha}\vec{\beta} + \vec{\alpha}\vec{\beta} - \vec{\beta}^2 = -2|\vec{\alpha}|^2 + 3\vec{\alpha}\vec{\beta} - |\vec{\beta}|^2 = -2 \cdot 2^2 + 3 \cdot 6 - (2\sqrt{3})^2 = -2$$

$$\gamma) |\vec{\gamma}|^2 = \vec{\gamma}^2 = (-2\vec{\alpha} + \vec{\beta})^2 = 4\vec{\alpha}^2 - 4\vec{\alpha}\vec{\beta} + \vec{\beta}^2 = 4 \cdot 4 - 4 \cdot 6 + 12 = 4. \text{ Επομένως } |\vec{\gamma}| = \sqrt{4} = 2$$

$$|\vec{\delta}|^2 = \vec{\delta}^2 = (\vec{\alpha} - \vec{\beta})^2 = \vec{\alpha}^2 - 2\vec{\alpha}\vec{\beta} + \vec{\beta}^2 = 4 - 2 \cdot 6 + 12 = 4. \text{ Επομένως } |\vec{\delta}| = \sqrt{4} = 2$$

$$\delta) \cos(\vec{\gamma}, \vec{\delta}) = \frac{\vec{\gamma} \cdot \vec{\delta}}{|\vec{\gamma}| \cdot |\vec{\delta}|} = \frac{-2}{2 \cdot 2} = -\frac{1}{2}. \text{ Οπότε } \cos(\vec{\gamma}, \vec{\delta}) = -\frac{1}{2} \Rightarrow (\vec{\gamma}, \vec{\delta}) = 120^\circ$$

5.23 2)

$$\alpha) \vec{\alpha} \cdot \vec{\beta} = |\vec{\alpha}| \cdot |\vec{\beta}| \cdot \cos(\vec{\alpha}, \vec{\beta}) = \sqrt{2} \cdot 1 \cdot \cos 45^\circ = \frac{\sqrt{2} \cdot \sqrt{2}}{2} = 1$$

$$\beta) \vec{\gamma} \cdot \vec{\delta} = (\vec{\alpha} - 2\vec{\beta})(-\vec{\alpha} + \vec{\beta}) = -\vec{\alpha}^2 + \vec{\alpha}\vec{\beta} + 2\vec{\alpha}\vec{\beta} - 2\vec{\beta}^2 = -|\vec{\alpha}|^2 + 3\vec{\alpha} \cdot \vec{\beta} - 2|\vec{\beta}|^2 = -\sqrt{2}^2 + 3 \cdot 1 - 2 \cdot 1 = -1$$

$$\gamma) |\vec{\gamma}|^2 = \vec{\gamma}^2 = (\vec{\alpha} - 2\vec{\beta})^2 = \vec{\alpha}^2 - 4\vec{\alpha} \cdot \vec{\beta} + 4\vec{\beta}^2 = \sqrt{2}^2 - 4 \cdot 1 + 4 \cdot 1 = 2$$

$$\text{Επομένως } |\vec{\gamma}| = \sqrt{2} = -|\vec{\alpha}|^2 + 3\vec{\alpha} \cdot \vec{\beta} - 2|\vec{\beta}|^2 = -\sqrt{2}^2 + 3 \cdot 1 - 2 \cdot 1 = -1$$

$$\text{Ακόμη } |\vec{\delta}|^2 = \vec{\delta}^2 = (-\vec{\alpha} + \vec{\beta})^2 = \vec{\alpha}^2 - 2\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = \sqrt{2}^2 - 2 \cdot 1 + 1^2 = 1$$

$$\text{Επομένως } |\vec{\delta}| = 1$$

$$\delta) \cos(\vec{\gamma}, \vec{\delta}) = \frac{\vec{\gamma} \cdot \vec{\delta}}{|\vec{\gamma}| \cdot |\vec{\delta}|} = \frac{-1}{\sqrt{2} \cdot 1} = -\frac{\sqrt{2}}{2} \quad \text{Επομένως } (\vec{\gamma}, \vec{\delta}) = 135^\circ$$

5.23 3)

$$\alpha) \vec{\alpha} \cdot \vec{\beta} = |\vec{\alpha}| \cdot |\vec{\beta}| \cdot \cos(\vec{\alpha}, \vec{\beta}) = 2 \cdot 2 \cdot \cos 60^\circ = 2 \cdot 2 \cdot \frac{1}{2} = 2$$

$$\beta) \vec{\gamma} \cdot \vec{\delta} = (\vec{\alpha} - \vec{\beta})(2\vec{\alpha} - \vec{\beta}) = 2\vec{\alpha}^2 - \vec{\alpha} \cdot \vec{\beta} - 2\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = 2|\vec{\alpha}|^2 - 3\vec{\alpha} \cdot \vec{\beta} + |\vec{\beta}|^2 = 2 \cdot 2^2 - 3 \cdot 2 + 2^2 = 6$$

$$\gamma) |\vec{\gamma}|^2 = \vec{\gamma}^2 = (\vec{\alpha} - \vec{\beta})^2 = \vec{\alpha}^2 - 2\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = 2^2 - 2 \cdot 2 + 2^2 = 4$$

$$\text{Επομένως } |\vec{\gamma}| = \sqrt{4} = 2$$

$$|\vec{\delta}|^2 = \vec{\delta}^2 = (2\vec{\alpha} - \vec{\beta})^2 = 4\vec{\alpha}^2 - 4\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = 4 \cdot 2^2 - 4 \cdot 2 + 2^2 = 12$$

$$\text{Επομένως } |\vec{\delta}| = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

$$\delta) \cos(\vec{\gamma}, \vec{\delta}) = \frac{\vec{\gamma} \cdot \vec{\delta}}{|\vec{\gamma}| \cdot |\vec{\delta}|} = \frac{6}{2 \cdot 2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\text{Επομένως } (\vec{\gamma}, \vec{\delta}) = \frac{\sqrt{3}}{2} \Rightarrow (\vec{\gamma}, \vec{\delta}) = 30^\circ$$

5.23 4)

$$\alpha) \vec{\alpha} \cdot \vec{\beta} = |\vec{\alpha}| \cdot |\vec{\beta}| \cdot \cos(\vec{\alpha}, \vec{\beta}) = 2 \cdot 4 \cdot \cos 120^\circ = 2 \cdot 4 \cdot \left(-\frac{1}{2}\right) = -4$$

$$\begin{aligned}\beta) \quad \vec{\gamma} \cdot \vec{\delta} &= (2\vec{\alpha} + \vec{\beta})(4\vec{\alpha} + \vec{\beta}) = 8\vec{\alpha}^2 + 2\vec{\alpha} \cdot \vec{\beta} + 4\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = \\ &= 8|\vec{\alpha}|^2 + 6\vec{\alpha} \cdot \vec{\beta} + |\vec{\beta}|^2 = 8 \cdot 2^2 + 6(-4) + 4^2 = 24\end{aligned}$$

$$\gamma) \quad |\vec{\gamma}|^2 = \vec{\gamma}^2 = (2\vec{\alpha} + \vec{\beta})^2 = 4\vec{\alpha}^2 + 4\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = 4 \cdot 2^2 + (-4) + 4^2 = 16$$

$$\text{Επομένως} \quad |\vec{\gamma}| = \sqrt{16} = 4$$

$$|\vec{\delta}|^2 = \vec{\delta}^2 = (4\vec{\alpha} + \vec{\beta})^2 = 16\vec{\alpha}^2 + 8\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = 16 \cdot 2^2 + 8(-4) + 4^2 = 48$$

$$\text{Επομένως} \quad |\vec{\delta}| = \sqrt{48} = 4\sqrt{3}$$

$$\delta) \quad \text{συν}(\vec{\gamma}, \vec{\delta}) = \frac{\vec{\gamma} \cdot \vec{\delta}}{|\vec{\gamma}| \cdot |\vec{\delta}|} = \frac{24}{4 \cdot 4\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\text{Επομένως} \quad \text{συν}(\vec{\gamma}, \vec{\delta}) = \frac{\sqrt{3}}{2} \Rightarrow (\vec{\gamma}, \vec{\delta}) = 30^\circ$$

5.23 5)

$$\alpha) \quad \vec{\alpha} \cdot \vec{\beta} = |\vec{\alpha}| \cdot |\vec{\beta}| \cdot \text{συν}(\vec{\alpha}, \vec{\beta}) = 2 \cdot 4 \cdot \text{συν} \frac{\pi}{3} = 2 \cdot 4 \cdot \frac{1}{2} = 4$$

$$\begin{aligned}\beta) \quad \vec{\gamma} \cdot \vec{\delta} &= (\vec{\alpha} - \vec{\beta})(2\vec{\alpha} + \vec{\beta}) = 2\vec{\alpha}^2 + \vec{\alpha} \cdot \vec{\beta} - 2\vec{\alpha} \cdot \vec{\beta} - \vec{\beta}^2 = \\ &= 2|\vec{\alpha}|^2 - \vec{\alpha} \cdot \vec{\beta} - |\vec{\beta}|^2 = 2 \cdot 2^2 - 4 - 4^2 = -12\end{aligned}$$

$$\gamma) \quad |\vec{\gamma}|^2 = \vec{\gamma}^2 = (\vec{\alpha} - \vec{\beta})^2 = \vec{\alpha}^2 - 2\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = 2^2 - 2 \cdot 4 + 4^2 = 12$$

$$\text{Επομένως} \quad |\vec{\gamma}| = \sqrt{12} = 2\sqrt{3}$$

$$|\vec{\delta}|^2 = \vec{\delta}^2 = (2\vec{\alpha} + \vec{\beta})^2 = 4\vec{\alpha}^2 + 4\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = 4 \cdot 2^2 + 4 \cdot 4 + 4^2 = 48$$

$$\text{Επομένως} \quad |\vec{\delta}| = \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$$

$$\delta) \quad \text{συν}(\vec{\gamma}, \vec{\delta}) = \frac{\vec{\gamma} \cdot \vec{\delta}}{|\vec{\gamma}| \cdot |\vec{\delta}|} = \frac{-12}{2\sqrt{3} \cdot 4\sqrt{3}} = -\frac{1}{2}$$

$$\text{Επομένως} \quad \text{συν}(\vec{\gamma}, \vec{\delta}) = -\frac{1}{2} \Rightarrow (\vec{\gamma}, \vec{\delta}) = \frac{2\pi}{3}$$