

5.22 1)

$$\cos(\vec{\alpha}, \vec{\beta}) = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}| \cdot |\vec{\beta}|} = \frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2} \cdot \sqrt{x_2^2 + y_2^2}} = \frac{-2 \cdot 0 + 2 \cdot 4}{\sqrt{(-2)^2 + 2^2} \cdot \sqrt{0^2 + 4^2}} = \frac{8}{\sqrt{8} \cdot 4} = \frac{\sqrt{8}}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\text{Οπότε } \cos(\vec{\alpha}, \vec{\beta}) = \frac{\sqrt{2}}{2} \Rightarrow \angle(\vec{\alpha}, \vec{\beta}) = 45^\circ$$

5.22 2)

$$\cos(\vec{\alpha}, \vec{\beta}) = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}| \cdot |\vec{\beta}|} = \frac{\sqrt{3} \cdot 1 + \sqrt{3} \cdot 1}{\sqrt{\sqrt{3}^2 + 1^2} \cdot \sqrt{\sqrt{3}^2 + 1^2}} = \frac{2\sqrt{3}}{2 \cdot 2} = \frac{\sqrt{3}}{2}$$

$$\text{Οπότε } \cos(\vec{\alpha}, \vec{\beta}) = \frac{\sqrt{3}}{2} \Rightarrow \angle(\vec{\alpha}, \vec{\beta}) = \frac{\pi}{6}$$

5.22 3)

$$\cos(\vec{\alpha}, \vec{\beta}) = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}| \cdot |\vec{\beta}|} = \frac{-2 \cdot \sqrt{3} + 0 \cdot 3}{\sqrt{(-2)^2} \cdot \sqrt{\sqrt{3}^2 + 3^2}} = \frac{-2\sqrt{3}}{2\sqrt{12}} = -\frac{1}{\sqrt{4}} = -\frac{1}{2}$$

$$\text{Οπότε } \cos(\vec{\alpha}, \vec{\beta}) = -\frac{1}{2} \Rightarrow \angle(\vec{\alpha}, \vec{\beta}) = \frac{2\pi}{3}$$

5.22 4)

$$\cos(\vec{\alpha}, \vec{\beta}) = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}| \cdot |\vec{\beta}|} = \frac{0(-2) + (-4) \cdot 0}{|\vec{\alpha}| \cdot |\vec{\beta}|} = 0$$

$$\text{Οπότε } \cos(\vec{\alpha}, \vec{\beta}) = 0 \Rightarrow \angle(\vec{\alpha}, \vec{\beta}) = \frac{\pi}{2}$$

5.22 5)

$$\cos(\vec{\alpha}, \vec{\beta}) = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}| \cdot |\vec{\beta}|} = \frac{(-1) \cdot 2 + (-3) \cdot 1}{\sqrt{1^2 + 3^2} \cdot \sqrt{2^2 + 1^2}} = \frac{-5}{\sqrt{10} \cdot \sqrt{5}} = -\frac{5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\text{Οπότε } \cos(\vec{\alpha}, \vec{\beta}) = -\frac{\sqrt{2}}{2} \Rightarrow \angle(\vec{\alpha}, \vec{\beta}) = \frac{3\pi}{4}$$