5.18 1)

Είναι $\vec{\alpha}^2=|\vec{\alpha}|^2=2^2=4$, $\vec{\beta}^2=|\vec{\beta}|^2=3^2=9$ και $\vec{\alpha}\cdot\vec{\beta}=|\vec{\alpha}|\cdot|\vec{\beta}|\text{συν}\left(\hat{\vec{\alpha},\vec{\beta}}\right)=2\cdot3\cdot\text{συν}60^o=6\cdot\frac{1}{2}=3$

Αρχικά θα υπολογίσουμε το $|3\vec{\alpha}+2\vec{\beta}|^2$

Είναι $|3\vec{\alpha}+2\vec{\beta}|^2=\left(3\vec{\alpha}+2\vec{\beta}\right)^2=9\vec{\alpha}^2+12\vec{\alpha}\vec{\beta}+4\vec{\beta}^2 \overset{\vec{\alpha}^2=4\ ,\ \vec{\beta}^2=9\ ,\ \vec{\alpha}\vec{\beta}=3}{=} 9\cdot4+12\cdot3+4\cdot9=108$

Επομένως $|3\vec{\alpha}+2\vec{\beta}|^2=108 \overset{|3\vec{\alpha}+2\vec{\beta}|\geq0}{\Rightarrow} |3\vec{\alpha}+2\vec{\beta}|=\sqrt{108}\Rightarrow|3\vec{\alpha}+2\vec{\beta}|=6\sqrt{3}$

5.18 2)

Είναι $\vec{\alpha}^2=|\vec{\alpha}|^2=1^2=1$, $\vec{\beta}^2=|\vec{\beta}|^2=6^2=36$ και $\vec{\alpha}\cdot\vec{\beta}=|\vec{\alpha}|\cdot|\vec{\beta}|\cdot\text{συν}\left(\hat{\vec{\alpha},\vec{\beta}}\right)=1\cdot6\cdot\text{συν}60=3$

Είναι $|2\vec{\alpha}-\vec{\beta}|^2=\left(2\vec{\alpha}-\vec{\beta}\right)^2=4\vec{\alpha}^2-4\vec{\alpha}\cdot\vec{\beta}+\vec{\beta}^2=4\cdot1-4\cdot3+36=28$

Επομένως $|2\vec{\alpha}-\vec{\beta}|^2=28\Rightarrow|2\vec{\alpha}-\vec{\beta}|=\sqrt{28}=\sqrt{4\cdot7}=2\sqrt{7}$

5.18 3)

Είναι $\vec{\alpha}^2=|\vec{\alpha}|^2=4^2=16$, $\vec{\beta}^2=|\vec{\beta}|^2=2^2=4$ και $\vec{\alpha}\cdot\vec{\beta}=|\vec{\alpha}|\cdot|\vec{\beta}|\cdot\text{συν}\left(\hat{\vec{\alpha},\vec{\beta}}\right)=4\cdot2\cdot\text{συν}120^o=-4$

Είναι $|2\vec{\alpha}-3\vec{\beta}|^2=\left(2\vec{\alpha}-3\vec{\beta}\right)^2=4\vec{\alpha}^2-12\vec{\alpha}\cdot\vec{\beta}+9\vec{\beta}^2=4\cdot16-12\cdot(-4)+9\cdot4=148$

Επομένως $|3\vec{\alpha}-\vec{\beta}|^2=148\Rightarrow|3\vec{\alpha}-\vec{\beta}|=\sqrt{148}=\sqrt{4\cdot37}=2\sqrt{37}$

5.18 4)

Είναι $\vec{\alpha}^2=|\vec{\alpha}|^2=2^2=4$, $\vec{\beta}^2=|\vec{\beta}|^2=3^2=9$ και $\vec{\alpha}\cdot\vec{\beta}=|\vec{\alpha}|\cdot|\vec{\beta}|\cdot\text{συν}\frac{\pi}{3}=2\cdot3\cdot\frac{1}{2}=3$

Είναι $|3\vec{\alpha}-\vec{\beta}|^2=\left(3\vec{\alpha}-\vec{\beta}\right)^2=9\vec{\alpha}^2-6\vec{\alpha}\cdot\vec{\beta}+\vec{\beta}^2=9\cdot4-6\cdot3+9=27$

Επομένως $|3\vec{\alpha}-\vec{\beta}|^2=27\Rightarrow|3\vec{\alpha}-\vec{\beta}|=\sqrt{27}=\sqrt{3\cdot9}=3\sqrt{3}$

5.18 5)

Είναι $\vec{\alpha}^2=|\vec{\alpha}|^2=1^2=1$ και $\vec{\beta}^2=|\vec{\beta}|^2=2^2=4$

Επίσης $\vec{\alpha}\perp\vec{\beta}$, άρα $\vec{\alpha}\cdot\vec{\beta}=0$

Είναι $|2\vec{\alpha}-3\vec{\beta}|^2=\left(2\vec{\alpha}-3\vec{\beta}\right)^2=4\vec{\alpha}^2-12\vec{\alpha}\cdot\vec{\beta}+9\vec{\beta}^2=4\cdot1-12\cdot0+9\cdot4=40$

Επομένως $|2\vec{\alpha}-3\vec{\beta}|^2=40\Rightarrow|2\vec{\alpha}-3\vec{\beta}|=\sqrt{40}=2\sqrt{10}$

5.18 6)

Είναι $\vec{\alpha}^2=|\vec{\alpha}|^2=2^2=4$, $\vec{\beta}^2=|\vec{\beta}|^2=\sqrt{3}^2=3$

και $\vec{\alpha}\cdot\vec{\beta}=|\vec{\alpha}|\cdot|\vec{\beta}|\cdot\text{συν}\left(\hat{\vec{\alpha},\vec{\beta}}\right)=2\cdot\sqrt{3}\cdot\text{συν}\left(\frac{5\pi}{6}\right)=2\cdot\sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2}\right)=-3$

Είναι $|3\vec{\alpha}-2\vec{\beta}|^2=\left(3\vec{\alpha}-2\vec{\beta}\right)^2=9\vec{\alpha}^2-12\vec{\alpha}\cdot\vec{\beta}+4\vec{\beta}^2=9\cdot4-12\cdot(-3)+4\cdot3=84$

Επομένως $|3\vec{\alpha}-2\vec{\beta}|^2=84\Rightarrow|3\vec{\alpha}-2\vec{\beta}|=\sqrt{84}=\sqrt{4\cdot21}=2\sqrt{21}$