ΛΥΣΗ

Έχουμε: $|\vec{\alpha}| = \dfrac{\sqrt{3}}{2}$ (1) , $|\vec{\beta}| = \dfrac{1}{2}$ (2) και $|3\vec{\alpha} + 2\vec{\beta}| = |\vec{\alpha} - 2\vec{\beta}|$(3).

α) Από την (3) $\Leftrightarrow |3\vec{\alpha} + 2\vec{\beta}|^2 = |\vec{\alpha} - 2\vec{\beta}|^2 \Leftrightarrow (3\vec{\alpha} + 2\vec{\beta})^2 = (\vec{\alpha} - 2\vec{\beta})^2 \Leftrightarrow$

$9\vec{\alpha}^2 + 12\vec{\alpha}\cdot\vec{\beta} + 4\vec{\beta}^2 = \vec{\alpha}^2 - 4\vec{\alpha}\cdot\vec{\beta} + 4\vec{\beta}^2 \Leftrightarrow 16\vec{\alpha}\cdot\vec{\beta} = -8\vec{\alpha}^2 \Leftrightarrow \vec{\alpha}\cdot\vec{\beta} = -\dfrac{1}{2}\vec{\alpha}^2 = -\dfrac{1}{2}|\vec{\alpha}|^2$.

Η τελευταία σχέση, λόγω της (1) δίνει: $\vec{\alpha}\cdot\vec{\beta} = -\dfrac{1}{2}\cdot\left(\dfrac{\sqrt{3}}{2}\right)^2 = = -\dfrac{1}{2}\cdot\dfrac{3}{4} = -\dfrac{3}{8}$.

β) Επίσης: $συν(\widehat{\vec{\alpha}, \vec{\beta}}) = \dfrac{\vec{\alpha}\cdot\vec{\beta}}{|\vec{\alpha}|\cdot|\vec{\beta}|}$ .

Η τελευταία ισότητα λόγω του ερωτήματος (α) και των (1), (2) δίνει:

$συν(\widehat{\vec{\alpha}, \vec{\beta}}) = \dfrac{-\frac{3}{8}}{\frac{\sqrt{3}}{2}\cdot\frac{1}{2}} = -\dfrac{3\cdot4}{8\cdot\sqrt{3}} = -\dfrac{3}{2\cdot\sqrt{3}} = -\dfrac{\sqrt{3}}{2}$, άρα $(\widehat{\vec{\alpha}, \vec{\beta}}) = \dfrac{5\pi}{6}$ ή $150^0$.