

Για κάθε $x \in [\alpha, \beta]$ έχουμε

$$\alpha \leq x \leq \beta \quad \overset{\text{g:φθίνουσα}}{\Rightarrow} \quad g(\alpha) \geq g(x) \geq g(\beta) \Rightarrow f(\alpha) - \alpha \geq f(x) - x \geq f(\beta) - \beta \quad \overset{\substack{f(\alpha)=\alpha \\ f(\beta)=\beta}}{\Rightarrow}$$

$$\Rightarrow \alpha - \alpha \geq f(x) - x \geq \beta - \beta \Rightarrow 0 \geq f(x) - x \geq 0 \Rightarrow f(x) - x = 0 \Rightarrow \boxed{f(x) = x}$$