

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

7.6 1)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \left(\frac{x^2}{x-1} - \frac{x^2}{x+1} \right) &= \lim_{x \rightarrow +\infty} \frac{x^{\cancel{2}} + x^2 - x^{\cancel{2}} + x^2}{x^2 - 1} = \lim_{x \rightarrow +\infty} \frac{2x^2}{x^2 - 1} = \\ &= \lim_{x \rightarrow +\infty} \frac{2x^{\cancel{2}}}{x^{\cancel{2}} \left(1 - \frac{1}{x^2} \right)} = \frac{2}{1-0} = \boxed{2} \end{aligned}$$

7.6 2)

$$\lim_{x \rightarrow +\infty} \left(\frac{x^3}{x^2 + 1} - x \right) = \lim_{x \rightarrow +\infty} \frac{x^{\cancel{3}} - x^{\cancel{3}} - x}{x^2 + 1} = \lim_{x \rightarrow +\infty} \frac{-x^{\cancel{3}}}{x^{\cancel{2}} \left(1 + \frac{1}{x^2} \right)} = \frac{-1}{(+\infty)(1+0)} = \boxed{0}$$

7.6 3)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \left(\frac{x^3}{x^2 - 4} - \frac{x^2}{x+2} \right) &= \lim_{x \rightarrow +\infty} \left[\frac{x^3}{(x-2)(x+2)} - \frac{x^2}{x+2} \right] = \\ &= \lim_{x \rightarrow +\infty} \frac{x^3 - x^2(x-2)}{(x-2)(x+2)} = \lim_{x \rightarrow +\infty} \frac{x^{\cancel{3}} - x^{\cancel{3}} + 2x^2}{x^2 - 4} = \lim_{x \rightarrow +\infty} \frac{2x^{\cancel{2}}}{x^{\cancel{2}} \left(1 - \frac{4}{x^2} \right)} = \frac{2}{1-0} = \boxed{2} \end{aligned}$$

7.6 4)

$$\begin{aligned} \lim_{x \rightarrow -\infty} \left(\frac{1}{x-1} + \frac{1}{x+2} - \frac{x^2+1}{x^2+x-2} \right) &= \\ &= \lim_{x \rightarrow -\infty} \left(\frac{x+2}{(x-1)(x+2)} + \frac{x-1}{(x-1)(x+2)} - \frac{x^2+1}{(x-1)(x+2)} \right) = \lim_{x \rightarrow -\infty} \frac{-x^2+2x}{(x-1)(x+2)} = \\ &= \lim_{x \rightarrow -\infty} \frac{-x^2+2x}{x^2+x-2} = \lim_{x \rightarrow -\infty} \frac{x^{\cancel{2}} \left(-1 + \frac{2}{x} \right)}{x^{\cancel{2}} \left(1 + \frac{1}{x} - \frac{2}{x^2} \right)} = \frac{-1+0}{1+0-0} = \boxed{-1} \end{aligned}$$