

Θέτουμε $h(x) = \frac{f(x)}{x}$, οπότε $\lim_{x \rightarrow -\infty} h(x) = 2$ (1) και $f(x) = x \cdot h(x)$ (2)

και $\varphi(x) = \frac{g(x)}{\eta\mu \frac{1}{x}}$, οπότε $\lim_{x \rightarrow -\infty} \varphi(x) = -3$ (3) και $g(x) = \varphi(x) \eta\mu \frac{1}{x}$ (4)

Επομένως

$$\begin{aligned} \lim_{x \rightarrow -\infty} [f(x)g(x)] &\stackrel{(2), (4)}{=} \lim_{x \rightarrow -\infty} \left[x \cdot h(x) \varphi(x) \eta\mu \frac{1}{x} \right] = \lim_{x \rightarrow -\infty} \left[h(x) \varphi(x) x \eta\mu \frac{1}{x} \right] \stackrel{(1), (3)}{=} \\ &= 2 \cdot (-3) \cdot 1 = \boxed{-6}^* \end{aligned}$$

$$* \text{ Είναι } \lim_{x \rightarrow -\infty} \left(x \eta\mu \frac{1}{x} \right) \stackrel{\substack{\text{θέτουμε } y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \\ \text{όταν } x \rightarrow -\infty \text{ τότε } y \rightarrow 0^-}}{=} \lim_{y \rightarrow 0^-} \left(\frac{1}{y} \eta\mu y \right) \stackrel{\lim_{y \rightarrow 0^-} \frac{\eta\mu y}{y} = 1}{=} 1$$